1.1 Introduction to Algebra

Need To Know

- What are Algebraic Expressions?
- Translating
  - Expressions
  - Equations

What is Algebra?

They say the only thing that stays the same is change. Our physical world is always changing and varying. In order to understand, interpret and predict the physical world we need a math way to express variableness – Algebra.

Algebra revolutionized the way we interact with the world.

Algebra is the power to translate the real world into mathematics.

This course will give the skills to exercise and understand this power.

Expressions

Definitions:

- A ___________ is a letter used to represent a number that can change or that is unknown.
- A ____________ is another name for a number.
- An _______________________ is a math statement with variables and/or numbers, often with operations signs and grouping symbols.

- Examples: w + 10, z/9, 2y(a + 3), 5, x

Evaluate the Expression

Evaluate means _______________________.

Evaluate the expressions below:

13 – z when z = 6

$$\frac{5z}{y} \quad when\ z = 9\ and\ y = 15$$

Translation

2 more than Bill's age.

4 less than d.

The sum of 7 and twice n

83% of the possible pts.

a + b	
a – b	
ab, a•b a(b)	
a÷b a/b $\frac{a}{b}$	

Equations

Definitions:

An ___________ is a math sentence that sets two expressions equal.

Examples:

2 + 7 = 9

5(4) = 10

x – 3 = 9

Equations

Definitions:

A __________ is a number for the variable that makes the equation true.

Examples:

Is 7 a solution to 94/y = 12?

Translate:

15% of all waste is recycled. This is the same as 47 million tons of recycled material. What's the total waste generated?

end

1.2 The Laws of Algebra

Need To Know

- Some of the Laws of Algebra
  - Commutative
  - Associative
  - Distributive

Commutative Law

Commutative Law of Addition

-
- Changing ________ in addition does not change the result.

Commutative Law of Multiplication

-
- Changing ________ in multiplication doesn't change the result.

Associative Law

Associative Law of Addition

-
- Changing _________ in addition does not change the result.

Associative Law of Multiplication

-
- Changing _______ in multiplication doesn't change the result.

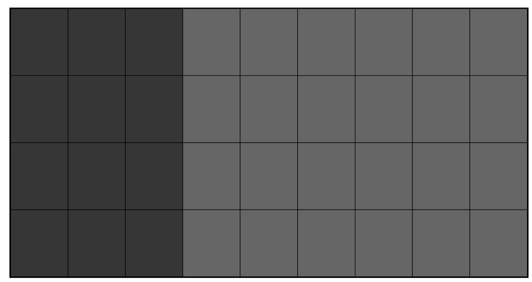
Distributive Law

Distributive Law

- a(b + c) =
- Multiplication distributes across addition
- Examples

Distributive Law

Distributive Law – a graphical look



Check for Understanding

Match the statement to its corresponding Law

Math Statement	Laws
x(9w) = (x9)w	Commutative
x + 9 + w = 9 + x + w	Associative
4(a – 5) = 4a – 20	Distributive

Check for Understanding

Identify which law(s) correspond to each statement

1. $t + (3 + w) = (3 + w) + t$

2. $(7 + y) + x = 7 + (x + y)$

Distributive Law – in reverse

Definition:

<u>Factoring or factor</u> means to write an expressions as a product.

The Distributive Law backwards: $ab + ac = a(b + c)$

Examples: Factor each expression.

$5y + 5z$

$9 + 9x$

$14a + 56b + 7$

end

1.3 Fractions

Need To Know

- Prime Factoring
- Operations on Fractions
  - Simplify (Reduce)
  - Multiply
  - Divide
  - Add
  - Subtract

Vocabulary

Definitions:

Prime numbers – are numbers that can only be factored by one and itself.

{

Prime factoring - means to write a number as a product of only prime numbers.

Prime Factoring

Prime factor 48

Prime factor 420

Reducing Fractions

Reduce $\frac{5}{15}$ $\frac{9}{21}$

Reduce $\frac{279}{310}$

Multiplication of Fractions

Simplify each expression

$$\frac{6}{5}\left(\frac{2}{7}\right)$$

$$\frac{2}{3}\cdot\frac{5}{x}$$

$$7\cdot\left(\frac{2}{9}\right)$$

Recall fraction multiplication

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

Division of Fractions

Recall – Division of fractions is the same as ______________________________.

Simplify:

$$\frac{7}{9}\div\left(\frac{1}{6}\right)$$

$$15\div\left(\frac{3}{2}\right)$$

end

1.3.B Add and Subtract Fractions

Need To Know

- How to add/subt. fractions with same denominator
- Rename fractions
- Find Least Common Denominator
- How to add/subt. fractions with different denominator

Add and Subtract Fractions

Recall the method to add fractions

Recall the method to subtract fractions

Renaming Fractions

Recall how to use the "fancy one" to rename fractions.

$\frac{9}{16} = \frac{}{48}$ $\frac{19}{42} = \frac{}{210}$

Least Common Denominator

Definition –

The <u>least common denominator (LCD)</u> for a set of denominators is the smallest multiple that each denominator can divide into evenly.

Examples: Find the LCD

$\frac{1}{6}, \frac{1}{10}$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}$

How to find the LCD

There are three ways to find LCD's

Use intuition
Use a list of multiples
Use the prime factoring method

How to find the LCD

Listing multiples method
Example: Find the LCD of 12 and 18

How to find the LCD

Prime factor method

Example: Find the LCD of 60 and 42.

Steps to find LCD

1. Prime factor each denominator
2. Create a product
   - using each factor
   - raised to the highest exponent that occurs in any one factoring

Add and Subtract Fractions

Recall how to add fraction

$$\frac{3}{10} + \frac{4}{14}$$

How to + or – fraction

Find the LCD

Rename each fraction (use the "fancy one")

Add numerators

Reduce

Practice

- **How to + or – fraction**
1. Find the LCD
2. Rename (use the "fancy one")
3. Add numerators
4. Reduce

Simplify:

$$\frac{3}{8} + \frac{2}{5} + \frac{1}{4}$$

Simplify:

$$\frac{1}{30} + \frac{9}{40}$$

Practice

- **How to + or – fraction**
1. Find the LCD
2. Rename (use the "fancy one")
3. Add numerators
4. Reduce

Simplify:

$$3 - \frac{3}{5}$$

Simplify:

$$\frac{23}{70} - \frac{29}{84}$$

end

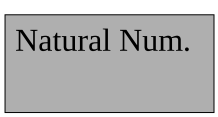
1.4 Subsets of Real Numbers

<u>Need To Know</u>

- Subsets of the Real Numbers
- Comparisons Symbols
- Absolute Value

Number Sets

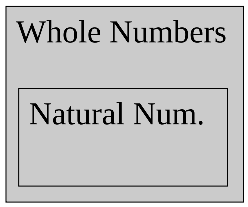
<u>Natural Numbers</u>
(Counting Numbers)
are the numbers =
{1, 2, 3, 4, 5, …}



Number Sets

<u>Whole Numbers</u> are the numbers =
{0, 1, 2, 3, 4, …}

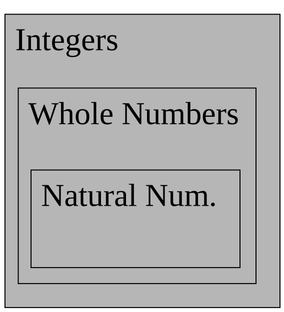
<u>Whole Numbers</u> =
{0} + Natural Numb.



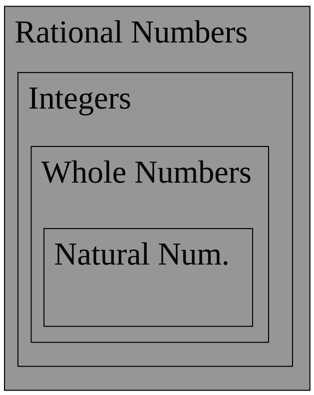
Number Sets

<u>Integers</u>
are the numbers =
{… -3, -2, -1, 0, 1, 2, 3, …}

<u>Integers</u> =
Negative Natural Numbers + Whole Numbers



Number Sets



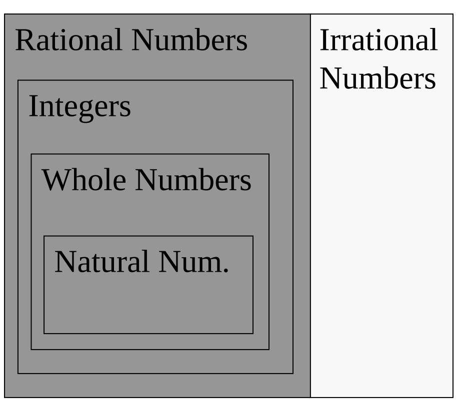
Rational Numbers

are the numbers of the form a/b where b is not zero.

Rational Numbers is the set of all fractions

Number Sets

REAL NUMBERS



Irrational Numbers

are the numbers that can not be written as fractions.

Real Numbers

are the collection of all Rational and Irrational Numbers.

Check for Understanding

$\left\{-5, -0.25, 0, 1, \pi, \frac{2}{7}, \sqrt{2}, 0.333, 5\right\}$

Categorize and list the numbers from the set to each set below.

Natural Numbers

Whole Numbers

Integers

Rational Numbers

Irrational Numbers

Real Numbers

Comparison Symbols

<u>True or False</u>

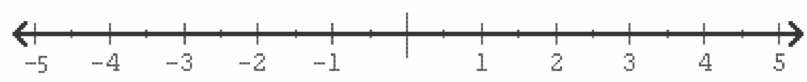
$4 \leq -4$

$-4 \leq 3$

$-4 \leq -4$

a = b	a is equal to b
a ≠ b	a is not equal to b
a < b	a is less than b
a ≤ b	a is less than **OR** equal to b
a > b	a is greater than b
a ≥ b	a is greater than **OR** equal to b

Number Line



<u>Key Vocabulary</u>

Positive Numbers – Numbers to the right of zero.

Negative Numbers – Numbers to the left of zero.

Opposite of a number is on the other side of zero.

Points on the number line correspond to real numbers.
All of the points represent all of the Real Numbers.

Absolute Value – The distance of a number from zero
(written as |x|)

Inequality Comparisons

If a number is further ____ on the number line, it is ____ than (<).

If a number is further ____ on the number line, it is __________ than (>).

Examples: Fill in the blank with < or >.

-7 __ 3   -¼ __ -¾ |-8| __ |-5|

end

1.5 Addition of Real Numbers

Need To Know

- Two models for addition
- Rules to add signed numbers
- Simplifying Expressions
- Translation

Two Models for Addition

About Addition

- The financial model for adding signed numbers.
- The vector model for adding signed number.
- Why do we have to look at two models?

Two Models for Addition

Why do we have to look at two models?

- Developing intelligence requires the ability to see things from more than one perspective.
- These models help us generalize the rules of adding.
-

Two Models for Addition

Why do we have to look at two models?

- Developing intelligence requires the ability to see things from more than one perspective.
- These models help us generalize the rules of adding.
-

Adding Signed Numbers

Financial Model

- ________ **numbers** correspond to deposits or credits to your account.
- ________ **numbers** correspond to withdrawals or debits from your account.

Deposit + Deposit =

Debt + Debt =

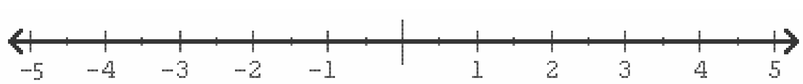
Deposit + Debt =

Debt + Deposit =

Adding Signed Numbers

Vector Model

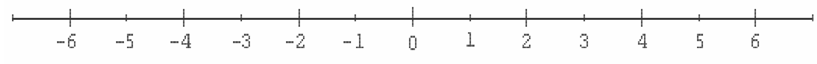
- **Vectors** are graphs of arrows with length and direction.
- **Positive numbers** are arrows to the right.
- **Negative numbers** are arrows to the left.



Examples:

1) 2 + 3   2) (-2) + (-3)   3) (-2) + 3   4) 2 + (-3)

Adding Signed Numbers-Rules



Rules for Adding

1) If the signs are the **same**, add values and keep the sign.

2) If the signs are the **opposite**, subtract values and keep the sign of larger the value.

- Examples

1) 2 + 5
2) (-3) + (-1)
3) (-5) + 2
4) 4 + (-6)

Like Terms

Definition

- ________ are parts of an algebraic expression separated by + or -. They may be numbers and/or variables often combined with multiplication or division.
- _____________________ is the number factor of a term.
- ________ are terms with the exact same variable factors.

Add & Subtract Like Terms

How can the Distributive Law help us make this expression simplify:

$4x + 9x$

Adding and Subtracting Like Terms is as simple as adding or subtracting coefficients.

Simplify each:

$-11b + 5b$

$2x + (-5y) + (-5x) + (-9y)$

$8 + a + (-5.5a) + 7$

Translation and Practice

Write the expression in mathematics and simplify.

The sum of –5 and –11 increased by 4.

Simplify the following expression.

$[18 + (-5)] + [9 + (-10)]$

1.6 Subtracting Signed Numbers

Need To Know

- Opposites
- Idea of Subtraction
- Rule for Subtraction
- Translation

Opposites

Definition:

The opposite (additive inverse) of a number "a" is written $-a$.

Recall: $-(-a) = a$

Example: Find $-x$ and $-(-x)$ when $x = 3$

The Law of Opposites

$a + (-a) = 0$

Idea of Subtraction

Goal:

- Take our common sense idea of distance to make the rule of subtraction understandable.

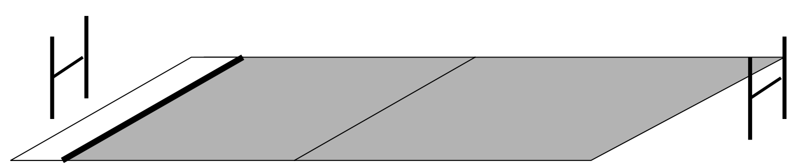
Problem:

- If a football team makes a play from the 33 yard line to the 39 yard line, how much distance did the team gain.
- Imagine a football team who punts the football from 2 yards behind the goal line. The ball stops on the 50 yard line. How many yards did the ball travel?

Idea of Subtraction

Our first problem:

Find the distance from 33 yard line to the 39 yard line.

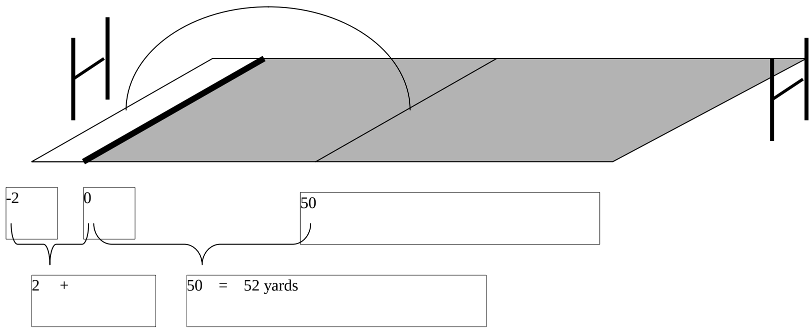


To find the distance between points we subtract:
Distance = the end point – the start point.

Idea of Subtraction

Our Second Problem:

Consider the distance on a picture.



Rule For Subtraction

<u>Rule for subtracting signed numbers</u>

- Subtraction is the same as adding the opposite.
- ______________________________

- <u>Examples:</u> Change each to addition
  - 3 – 4
  - 3 – (-4)
  - -3 – 4
  - -3 – (-4)

Subtraction of Signed Numbers

Write each as an addition problem and the simplify your answer.

11 – 5

11 – (-5)

-11 – 5

-11 – (-5)

Subtraction Practice

Simplify	Simplify
9 – 4 – 5	-9x + 5 – 3x
24 – (-12) + 7 – 15	-5 + 3b – 7 – 5b

Translation

Difference, decreased, take away, reduced, less and **<u>from</u>** are all key words for subtraction.

Examples: Translate into mathematics and use the rule of subtraction to simplify.

Subtract 5 from 8.

Find the difference of 4 and –7.

1.7 Mult. & Div. of Real Numbers

<u>Need To Know</u>

- Multiplication of Signed Numbers
- Division of Signed Numbers
  - Apply to: Integers, Decimals and Fractions

Sign Patterns in Multiplication

Look at these multiplication problems and draw conclusions about sign results.

(3)(2) =	3(-2) =
(3)(1) =	2(-2) =
(3)(0) =	1(-2) =
(3)(-1) =	0(-2) =
(3)(-2) =	-1(-2) =

Practice - Multiplication

Summary of sign pattern for multiplication

(+)(+) = ___ (+)(–) = ___

(–) (–) = ___ (–)(+) = ___

Simplify each expression

(-4)(-8)(-1) -3·(-5)·(-2)·(-1)

The product of an odd # of negatives is ___

The product of an even # of negatives is ___

Practice - Multiplication

Simplify each expression

Recall fraction multiplication

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$$

$$-9\left(\frac{1}{3}\right)$$

$$-\frac{6}{5}\left(-\frac{2}{7}\right)$$

Division with Signed Numbers

Recall: $a \div b = a \cdot \frac{1}{b} = \frac{a}{b}$

Sign rules for division work just as multiplication.

- (+) ÷ (+) = (+)
- (−) ÷ (−) = (+)
- (+) ÷ (−) = (−)
- (−) ÷ (+) = (−)

Fraction Facts

$\frac{\text{Top}}{\text{Bottom}} = \frac{\text{Numerator}}{\text{Denominator}}$

$$\frac{0}{5} \qquad \frac{5}{0}$$

Division of Fractions

Recall – Division of fractions is the same as multiplication by the reciprocal.

$$\frac{\mathbf{a}}{\mathbf{b}} \div \frac{\mathbf{c}}{\mathbf{d}} = \frac{\mathbf{a}}{\mathbf{b}} \cdot \frac{\mathbf{d}}{\mathbf{c}}$$

$$-\frac{7}{9} \div \left(\frac{1}{6}\right) \qquad -15 \div \left(-\frac{3}{2}\right)$$

1.8 Exponents and Order of Op.

Need To Know

- Exponents
- Order of Operations
- Simplifying Expressions

Exponents

Exponents mean ______________________.

Notation: 4^3

Examples:

5^4

$(-7)^2$

$(2x)^5$

-3^2

Practice - Order of Operation

Simplify:

$20 \div 5 + 15$

$8 \div 2 \cdot 4$

$12 \div (-3 - 5)$

Order of Operations –
Always work left to right

1.
2.
3.
4.

Practice - Order of Operation

Simplify:

$-2(6-10) - 3|5-8|$

Order of Operations –
Always work left to right

1. Evaluate grouped expressions.
2. Evaluate exponents.
3. Evaluate multiplication and division in the order they appear.
4. Evaluate addition and subtraction in the order they appear.

Practice - Order of Operation

Simplify:

$$-2\cdot 5^2+3\cdot 2^3\div(-1)^4$$

Order of Operations –

Always work left to right

1. Evaluate grouped expressions.
2. Evaluate exponents.
3. Evaluate multiplication and division in the order they appear.
4. Evaluate addition and subtraction in the order they appear.

Practice - Order of Operation

Simplify:

$$\frac{6(-2)+5(-3)}{5(4)-11}$$

Order of Operations –

Always work left to right

1. Evaluate grouped expressions.
2. Evaluate exponents.
3. Evaluate multiplication and division in the order they appear.
4. Evaluate addition and subtraction in the order they appear.

Simplifying Expressions

Recall: $-1(a) = -a$, and that
opposite and negative are synonymous
What is $-(a + b) =$

Examples:

$-(7z + 6)$

$-(13y - 5x + 8)$

$-(-8x^3 + 4x^2 - 3x)$

Simplifying Expressions

Examples:

$7y - (2y + 9)$

$9t - 5r - 2(3r + 6t)$

$8n^2 + n - 7(n + 2n^2)$