



6.1 Reducing Rational Expressions

Need To Know



- Review reducing fraction
- Review polynomial factoring methods
- Idea of rational expressions w/ restrictions
- How to reduce rational expressions



Review Reducing Fraction

Reduce $18/24$

1. By shortcut

2. Show all steps

How to Reduce

1. _____

2. _____

Review Factoring Polynomials

- A. Check for GCF on all factoring (Always do first!)
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas: $x^2 + 2xy + y^2 = (x + y)^2$
 $x^2 - 2xy + y^2 = (x - y)^2$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely

Idea of Rational Expressions

A rational expression is a fancy word for a polynomial fraction.

$$\frac{4}{5}$$

$$\frac{4}{x}$$

$$\frac{x}{x-3}$$

$$\frac{x+1}{x^2+x-2}$$

| x | $\frac{4}{x}$ |
|----|---------------|
| -2 | |
| 0 | |
| 1 | |
| 3 | |

| x | $\frac{x}{x-3}$ |
|----|-----------------|
| -2 | |
| 0 | |
| 1 | |
| 3 | |

| x | $\frac{x+1}{x^2+x-2}$ |
|----|-----------------------|
| -2 | |
| 0 | |
| 1 | |
| 3 | |



Rational Expressions

Rational expression, algebraic fractions and fractional expressions all mean the same.

We must be careful _____, so we sometimes have to put restrictions on x.

$$\frac{x+5}{x-3}$$

$$\frac{6}{x^2 + x - 6}$$



Reducing Rational Expressions

Reduce to lowest terms

$$\frac{45x^2y^3}{9x^5y}$$

$$\frac{4x-12}{6x}$$

How to Reduce

- 1.
- 2.



Reducing Rational Expressions

Reduce to lowest terms

$$\frac{5a+15}{10a^2-90}$$

How to Reduce

- 1.
- 2.



Reducing Rational Expressions

Reduce to lowest terms

$$\frac{x^2-4}{x^2-2x-8}$$

How to Reduce

1. Factor
2. Divide common factors



Reducing Rational Expressions

Reduce to lowest terms

$$\frac{x-3}{3-x}$$

$$\frac{7a^2 - 7b^2}{3b^2 - 3a^2}$$

How to Reduce

1. Factor
2. Divide common factors





6.2 Rational Expression

Multiplication
& Division

Need To Know



- Review multiplication and division of fractions
- Multiplying and dividing rational expressions



Multiplying Fractions - Review

Examples

$$\frac{10}{21} \cdot \frac{6}{25}$$

Recall:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a c}{b d}$$

Multiply Factions

1. _____
 - a) _____
 - b) _____
2. _____



Canceling and Reducing Fractions

Reduce each:

$$\frac{4}{4} \quad \frac{4}{12} \quad \frac{a}{3 \cdot a} \quad \frac{x+6}{x+6} \quad \frac{6}{x+6}$$



Multiply Rational Expressions

Multiply

$$\frac{2a+10}{a^3} \cdot \frac{a^2}{3a+15}$$

Multiply Fractions

1. Reduce
 - a) Factor
 - b) Divide common factors
2. Multiply/simplify



Multiply Rational Expressions

Multiply

$$\frac{x-5}{x+2} \cdot \frac{x^2-4}{3x-15}$$

Multiply Fractions

1. Reduce
 - a) Factor
 - b) Divide common factors
2. Multiply/simplify



Multiply Rational Expressions

Multiply

$$\frac{x^2+5x+4}{x^2-6x+8} \cdot \frac{x^2+5x-14}{x^2+8x+7}$$

Multiply Fractions

1. Reduce
 - a) Factor
 - b) Divide common factors
2. Multiply/simplify



Dividing Fractions - Review

Examples

$$\frac{9}{7} \div \frac{6}{35}$$

Recall:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

Divide Fractions

1.

2.



Divide Rational Expressions

Divide

$$\frac{t-3}{t+2} \div \frac{4t-12}{t-1}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$



Divide Rational Expressions

Divide

$$\frac{x^2 - x - 12}{x^2 - 16} \div \frac{x^2 + 6x + 9}{2x + 8}$$

end



6.3 Rational Expressions Addition & Subtraction

Need To Know



- Review Addition and Subtraction of Fraction with Same Denominator.
- Find the Least Common Multiple for Polynomial Expressions (same as LCD)

Add and Subtract Fractions

$$\frac{7}{20} - \frac{3}{20}$$

Recall:

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

$$\frac{2}{z} + \frac{3}{z}$$



Add and Subtract Fractions

$$\frac{y^2 + 6y}{y + 2} + \frac{2y + 12}{y + 2}$$

$$\frac{x - 7}{x^2 + 3x - 4} - \frac{2x - 3}{x^2 + 3x - 4}$$



Finding the LCD

$$\frac{5}{18} + \frac{7}{24}$$

There are three ways to find LCD's

1. Use intuition
2. Use a list of multiples
3. Use the prime factoring method



Least Common Denominator

Find the LCD:

18 and 24

$6a^2b^7$ and $9a^5b^2$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations



Least Common Denominator

Find the LCD:

$y^3 - y^2$ and $y^4 - y^2$

$2x^2 + 5x + 2$ and $2x^2 - x - 1$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations

end

6.4 Rational Expressions Addition & Subtraction

Need To Know



- Review addition and subtraction of fraction
- Add and subtract rational expression without the same denominator

Add & Subtract Fract.- Review

$$\frac{1}{21} + \frac{2}{9}$$

$$\frac{2}{x} - \frac{1}{3}$$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations

How to + or - fraction

1. _____
2. _____
3. _____
4. _____



Add & Subtract Rat. Expression

Add $\frac{7}{3x+6} + \frac{x}{x+2}$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations

How to + or – fraction

1. Find the LCD
2. Rename each fraction (use the “fancy one”)
3. Add numerators
4. Reduce



Add & Subtract Rat. Expression

Add $\frac{1}{x-3} + \frac{-6}{x^2-9}$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations

How to + or – fraction

1. Find the LCD
2. Rename each fraction (use the “fancy one”)
3. Add numerators
4. Reduce



Add & Subtract Rat. Expression

Add $\frac{5}{x^2 - 7x + 12} + \frac{1}{x^2 - 9}$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations

How to + or – fraction

1. Find the LCD
2. Rename each fraction (use the “fancy one”)
3. Add numerators
4. Reduce



Add & Subtract Rat. Expression

Subt. $\frac{x}{x^2 + 5x + 6} - \frac{2}{x^2 + 3x + 2}$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations

How to + or – fraction

1. Find the LCD
2. Rename each fraction (use the “fancy one”)
3. Add numerators
4. Reduce



Add & Subtract Rat. Expression

Add: $\frac{m+2}{m-7} + \frac{3-m}{49-m^2}$

Find LCD by Factoring

- Prime factor each denominator
- Multiply together each factor that occurs
- Use the greatest exponent in any one of the factorizations

How to + or - fraction

1. Find the LCD
2. Rename each fraction (use the "fancy one")
3. Add numerators
4. Reduce

end





6.5 Complex Fractions

Need To Know



- What are complex fractions?
- Two methods to simplify complex fractions



Complex Fractions

Complex fractions are fractions of fractions.
There are messy ones and really messy ones.

Examples:

$$\frac{\frac{2}{3}}{\frac{4}{5}} \quad \frac{\frac{4}{x} - \frac{1}{x^2}}{\frac{2}{x^2}} \quad \frac{3 + \frac{1}{5}}{1 - \frac{3}{5}} \quad \frac{\frac{5}{4x^3} - \frac{3}{8x}}{\frac{3}{2x} - \frac{3}{4x^3}}$$



Complex Fractions

What is another interpretation of fractions?

$$\frac{\frac{2}{3}}{\frac{4}{5}}$$

Recall Division:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$



Complex Fractions – Method #1

Simplify

$$\frac{\frac{4}{x} - \frac{1}{x^2}}{\frac{2}{x^2}}$$

Method #1

(Single fraction in denominator)

- 1)
- 2)

Complex Fractions – Method #2

Simplify

$$\frac{3 + \frac{1}{5}}{1 - \frac{3}{5}}$$

Method #2

(Two or more terms in denom.)

- 1)
- 2)

Complex Fractions – Method #2

Simplify

$$\frac{\frac{5}{4x^3} - \frac{3}{8x}}{\frac{3}{2x} - \frac{3}{4x^3}}$$

Method #2

(Two or more terms in denom.)

- 1) Clear all denominators
- 2) Multiply by a “fancy one” of the LCD of all fractions



Complex Fractions – Method #2

Simplify

$$\frac{x - 2 + \frac{1}{x}}{x - 5 + \frac{4}{x}}$$

Method #2

(Two or more terms in denom.)

- 1) Clear all denominators
- 2) Multiply by a “fancy one” of the LCD of all fractions



6.5 Conclusion

Method #1

(Single fraction in denominator)

- 1) See fractions a division
- 2) Change to multiplication by the reciprocal

Method #2

(Two or more terms in denom.)

- 1) Clear all denominators
- 2) Multiply by a “fancy one” of the LCD of all fractions



6.6 Solving Rational Equations

Need To Know



- Review Process for Solving Equations
- Apply to Equations of Rational Expressions



Review Process to Solve

Solve for x:

$$\frac{4}{5} - \frac{2}{3} = \frac{x}{9}$$

Recall Steps to Solve

Simplify (See 2.2)

1. Clear fractions
2. Clear parentheses
3. Clear like terms

Use Add Property

1. Get variable on one side
2. Get constant terms on the other side

Use Multiplication Property

1. Get unknown by itself

Check



Solving Rational Equations

Solve for x:

$$x + \frac{6}{x} = -7$$

Solve Rational Equations

List any restrictions

Simplify

Clear fractions by multiplying everything by the LCD/1

Solve New Equation

Check



Solving Rational Equations

Solve for x:

$$\frac{5}{x-1} = \frac{3}{x+2}$$

Solve Rational Equations

List any restrictions

Simplify

Clear fractions by multiplying everything by the LCD/1

Solve New Equation

Check



Solving Rational Equations

Solve for x:

$$\frac{x + 2}{5} - 1 = \frac{x - 2}{4}$$

Solve Rational Equations

List any restrictions

Simplify

Clear fractions by multiplying everything by the LCD/1

Solve New Equation

Check



Solving Rational Equations

Solve for x:

$$\frac{2}{t - 9} = \frac{t - 7}{t - 9}$$

Solve Rational Equations

List any restrictions

Simplify

Clear fractions by multiplying everything by the LCD/1

Solve New Equation

Check

end



6.7 Proportions

Need To Know



- Special Focus on Proportions
Page 380-382, Homework #23, 35–45 odd
- Idea of proportions
- How to solve proportion
- Solving proportion word problems



Ratios and Proportions

A _____ is a way to compare two numbers.

We write a ratio of a and b as: a to b or $\frac{a}{b}$

A _____ is an equation of two ratios.

Examples:

$$\frac{12}{36} = \frac{1}{3}$$

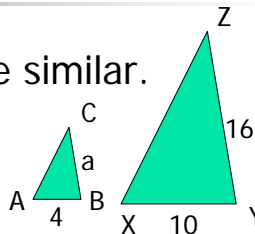
$$\frac{2}{9} = \frac{8}{x}$$

$$\frac{3}{x} - 1 = \frac{2}{x^2}$$



Apply Proportions

Triangles ABC and XYZ are similar.
Solve for side "a".



Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model



Apply Proportions

To estimate the number of trout in a lake, a naturalist catches, tags and releases 112 trout. Later, 82 trout are caught; 32 of the have tags. Estimate the number of trout in the lake.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model



Apply Proportions

A manufacturer knows that during production, 8 out of 100 parts made are defective. If they plan to produce 1,650 parts, how many can they expect to be defective.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

end