8.1 Definitions and Roots

- <u>Need To Know</u>
  - How to find the root of a number
  - Categorizing roots
  - How to find the root of an expression
  - How to solve application problems involving roots

Idea of Square Roots

In mathematics, once we learn an operation, we also learn the reverse of that operation.

For real numbers x and y,

If $y = x^2$, then x = ___

Don't Forget

Radicand

Radical Sign

Radical

Definitions and Roots

Definition – If x is any positive real number, then

$\sqrt{x}$ is the ________ square root of x and

$-\sqrt{x}$ is the ________ square root of x.

Examples:

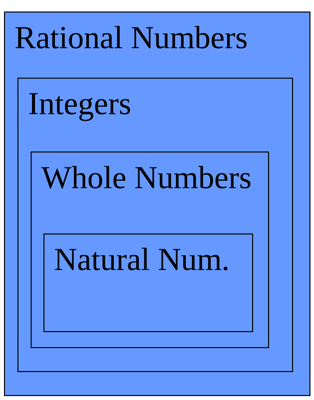
Name the two square roots of 81.

Find the root of $\sqrt{25}$

Simplify $-\sqrt{100}$

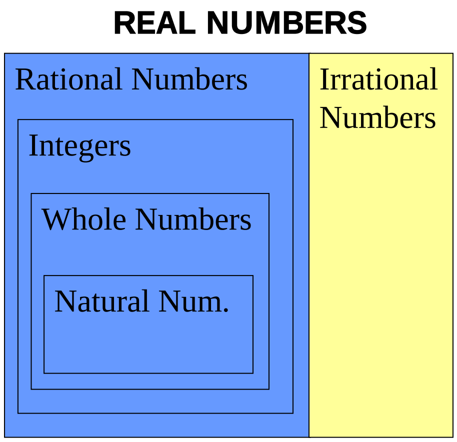
Evaluate $\sqrt{121}$

Recall Number Sets



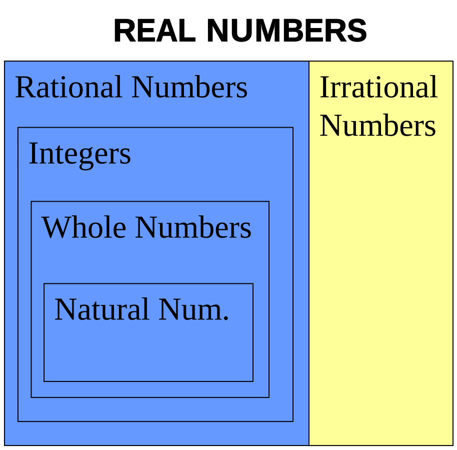
- Rational Numbers are the numbers of the form a/b where b is not zero.
- Rational Numbers is the set of all fractions

Recall Number Sets



- Irrational Numbers are the numbers that can not be written as fractions
- Real Numbers are the collection of all Rational and Irrational Numbers.

Categorize Roots



$\sqrt{21}$

$\sqrt{196}$

$\sqrt{40}$

$\sqrt{-9}$

Square Roots & Absolute Value

Consider $\sqrt{(-7)^2} = \sqrt{49} = 7$ and $\sqrt{(7)^2} = \sqrt{49} = 7$

Both come out positive.

Recall: |-7| = 7 and |7| = 7.

For all real numbers A, ______________________

Examples:

$\sqrt{y^2}$ $\sqrt{(5pq)^2}$ $\sqrt{\dfrac{100z^6}{36m^{14}}}$

Application

The speed of sound in feet per second, V, traveling through air with a temperature of ***t*** is given by the formula below. Find the speed of sound when the temperature is 5° C.

$$V = \frac{1087\sqrt{273+t}}{16.52}$$

8.1 Conclusion

- The square root is the number that "undoes" the square.
- Square roots can be positive or negative.
- The square root of zero is zero.
- The square root of a negative number is not a Real Number
- A square root results in a
  Rational, Irrational or Non-Real Number

8.2 Multiply and Simplify Radicals

- <u>Need To Know</u>
  - Multiplying of radicals
  - Simplifying radical expressions
    1. With numbers
    2. With variables

Multiplication Property of Radicals

- If A and B are real numbers ($\geq$ 0), then

____

Simplify Radical Expressions

- Simplify:

$\sqrt{12}$

$\sqrt{50}$

Perfect Square
1
4
9
16
25
36
49
64
81
100
121
144

Simplify Radical Expressions

- Simplify:

$\sqrt{288y}$

$\sqrt{125a^2}$

Perfect Square
1
4
9
16
25
36
49
64
81
100
121
144

Simplify Radical Expressions

- Simplify:

$\sqrt{x^3}$

$\sqrt{x^4}$

$\sqrt{x^5}$

Square roots undo squares

$\sqrt{x^2} = x$

$\sqrt{x^4} = x^2$

$\sqrt{x^6} = x^3$

$\sqrt{x^8} = x^4$

Practice Simplifying Radicals

- Simplify:

$\sqrt{36a^3}$ $\sqrt{50y^4}$ $4\sqrt{18xy^2}$

Practice Simplifying Radicals

- Simplify:

$\sqrt{6}\cdot\sqrt{18}$ $\sqrt{3xa}\cdot\sqrt{3xb}$ $\sqrt{50ab}\cdot\sqrt{10a^2b^7}$

end

8.3 Properties of Radicals

- Need To Know
  - Quotient Rule for Square Roots
  - Simplifying radical expressions
    1. With Fractions
    2. By Rationalizing the Denominator

Quotient Rule for Square Roots

- If A and B are real numbers ($B \neq 0$), then

Simplify Radical Expressions

- Simplify:

$$\frac{\sqrt{18}}{\sqrt{32}} \qquad \frac{\sqrt{35t^{11}}}{\sqrt{5t^5}}$$

Simplify Radical Expressions

- Simplify:

$$-\sqrt{\frac{25}{64}} \qquad \sqrt{\frac{10z^9}{18z^5}}$$

The Idea of a Simplified Radical

Which fraction is the simplest?

$$\frac{\sqrt{5}}{\sqrt{3}} = \frac{2.236067977...}{1.732050808...}$$

$$\frac{5}{\sqrt{3}} = \frac{5}{1.732050808...}$$

$$\frac{\sqrt{5}}{3} = \frac{2.236067977...}{3}$$

Rationalizing Denominators

- Goal: Change the fraction to make the denominator come out “nice”.

$$\frac{\sqrt{5}}{\sqrt{3}} =$$

Guidelines for Simplification

1. Remove ______________________________
2. Remove ______________________________
3. Remove ______________________________

- Examples:

$$\frac{7}{\sqrt{2}} \qquad\qquad \frac{\sqrt{4}}{\sqrt{27}}$$

Guidelines for Simplification

1. Remove fractions from radicals
2. Remove perfects for radicals
3. Remove radicals from denominators

- Examples:

$$\sqrt{\frac{5}{x}} \qquad\qquad \sqrt{\frac{7t^3}{32t}}$$

end

8.4 Operations with Radicals

- Need To Know
  - Recall Like Terms
  - Recall Radical Simplification
  - Add and Subtract Radicals
  - More Multiplication of Radical
  - More Rationalizing Denominators

Recall Simplify Radicals

- Recall properties and Perfects

1. $\sqrt{xy} = \sqrt{x}\sqrt{y}$

2. $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

$\sqrt{48}$

Perfect Square
1
4
9
16
25
36
49
64
81
100
121
144

Recall Like Terms

- Recall:
- ab + ac

Simplify:

4x + 7x

Simplify:

$4\sqrt{2}+7\sqrt{2}$

Add and Subtract Radicals

- Simplify each:

$5\sqrt{2}-8\sqrt{2}$

$9\sqrt{3}-4\sqrt{3}+\sqrt{3}$

$3\sqrt{7}+2\sqrt{3}$

$12\sqrt{14y}-\sqrt{14y}$

Practice

- Simplify Each

$3\sqrt{12}+5\sqrt{48}$

$4\sqrt{18}+\sqrt{32}-\sqrt{2}$

Multiplication

- Multiply each:

$\sqrt{5}\left(\sqrt{2}+\sqrt{15}\right)$

$\left(\sqrt{3}+5\right)\left(\sqrt{3}+2\right)$

$\left(\sqrt{x}+8\right)\left(\sqrt{x}-6\right)$

Multiplication

- Multiply each:

$$\left(\sqrt{5}-4\right)^2$$

$$\left(\sqrt{x}+\sqrt{7}\right)\left(\sqrt{x}-\sqrt{7}\right)$$

Remember: $(x+y)(x-y) = x^2 - y^2$

Division of Radical Expression

- Recall rationalizing denominators.
- Goal: Change the fraction to make the denominator come out "nice".

$$\frac{\sqrt{5}}{\sqrt{3}}$$

$$\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

Division of Radical Expression

- Recall rationalizing denominators.
- Goal: Change the fraction to make the denominator come out "nice".

$$\frac{\sqrt{5}}{\sqrt{5}-\sqrt{3}}$$

Remember: $(x+y)(x-y) = x^2 - y^2$

Practice

- Rationalize the denominator in each:

$$\frac{2}{3-\sqrt{7}}$$

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

end

8.5 Equations Involving Radicals

- <u>Need To Know</u>
  - The idea of solving radical equations
  - The Principle of Squaring

Idea for Solving Radical Equations

- Recall that square roots undo squares.
- Then how do you undo a square root?

The goal of solving equations requires us

____

______________________________________.

Solving Radical Equations

- Solve:

$$\sqrt{x+2} = 5$$

Idea for Solving Radical Equations

- We know from
- exponents that
- If a = b,
- Then $a^2 = b^2$
- but ...

Consider the equation

x = 5 original eq.

square both

new eq.

Square Property of Equality

We can square both sides of an equation, but ... we must check all solutions in the original equation.

Solving Radical Equations

- Solve:

$$\sqrt{3x+1} = -4$$

Solving Radical Equations

- Solve:

$$\sqrt{2a-3} + 4 = 9$$

Solving Radical Equations

- Solve:

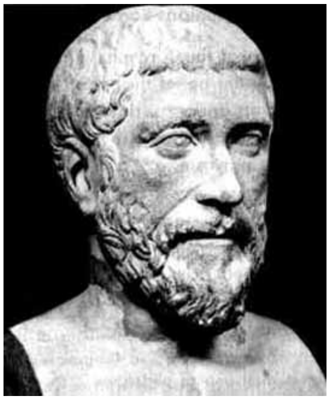
$$\sqrt{7-3x} = \sqrt{12+x}$$

end

8.6 Application - Right Triangles

- <u>Need To Know</u>
  - The Pythagorean Theorem
  - Apply to square root equations

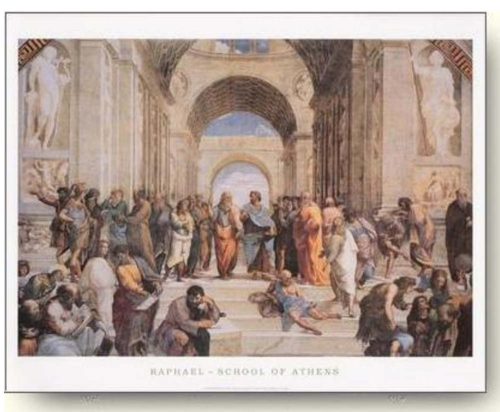
Pythagoras and The Theorem



Pythagoras and the Pythagoreans

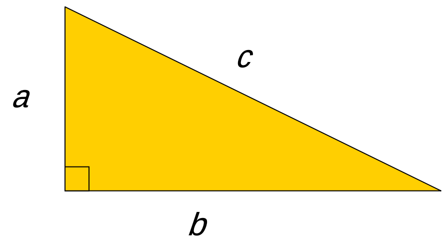
Pythagoras of Samos (c.560-c.480BC), mathematician, philosopher and religious leader, founded a religious community (the Pythagorean Order) in Croton on the coast of Italy around 530 BC. According to Aristotle (see *Metaphysics* 985b-986a), the Pythagoreans, first to develop the science of mathematics, revered number as the first principle of all things, probably due to their discovery that the principles of musical harmony could be explained with mathematics. http://plato.evansville.edu/

Pythagoras and The Theorem



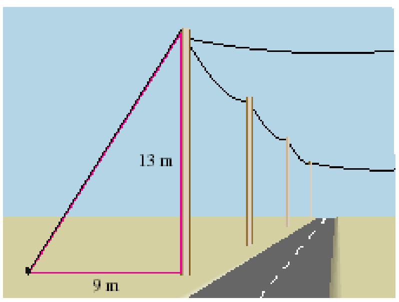
Pythagoras and The Theorem

Pythagorean Theorem:
In any right triangle,
if *a* and *b* are the lengths of the legs and
c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.



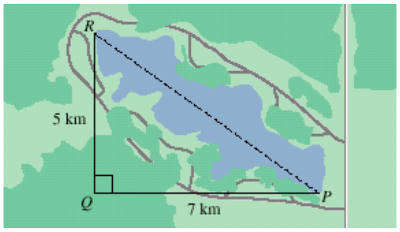
See SketchPad proof by Leonardo Da Vinci

Applications



How long does a guy wire need to be to secure the pole?

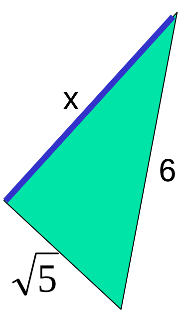
Applications



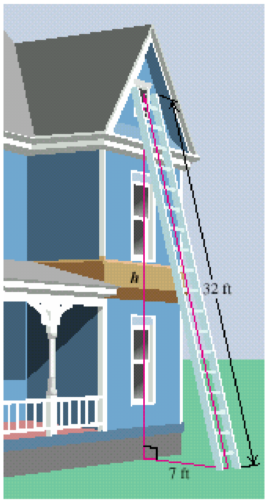
How wide is the lake?

Applications

Find the missing side



Applications



How high is it to the attic window?

end

8.7 Higher Roots

- <u>Need To Know</u>
  - Idea of Higher Roots
  - Product and Quotient Rules
  - Simplify Higher Roots

Idea of Higher Roots

You undo a second power or square with _________?

You undo a third power or cube with _________?

You undo a fourth power with _________?

You undo a fifth power with _________?

Notation and the nth Root

Note:
Index,
radicand

If c^3 = a, then $\sqrt[3]{a} =$

If c^n = a, then $\sqrt[n]{a} =$

Facts:

If n is even, there are two nth roots.

The positive one is ______

Even roots of negative numbers are not real numbers.

Notation and the nth Root

Examples:

$-\sqrt[4]{81}$

$\sqrt[3]{-125}$

$\sqrt[4]{-81}$

$\sqrt[5]{32}$

Perfects to Memorize

Perfect Square	Perfect Cubes	Perfect Fourths	Perfect Fifths
1	1	1	1
4	8	16	32
9	27	81	243
16	64	256	
25	125	625	
36	216		
49			
64			
81			
100			
121			
144			

The Product and Quotient Rules

$$\sqrt[n]{AB} = \sqrt[n]{A} \cdot \sqrt[n]{B} \qquad \sqrt[n]{\frac{A}{B}} = \frac{\sqrt[n]{A}}{\sqrt[n]{B}}$$

The Product and Quotient Rules

Examples:

$\sqrt[3]{32}$ $\qquad$ $\sqrt[5]{160}$

$\sqrt[3]{\frac{10}{27}}$ $\qquad$ $\sqrt[5]{\frac{17}{32}}$

end