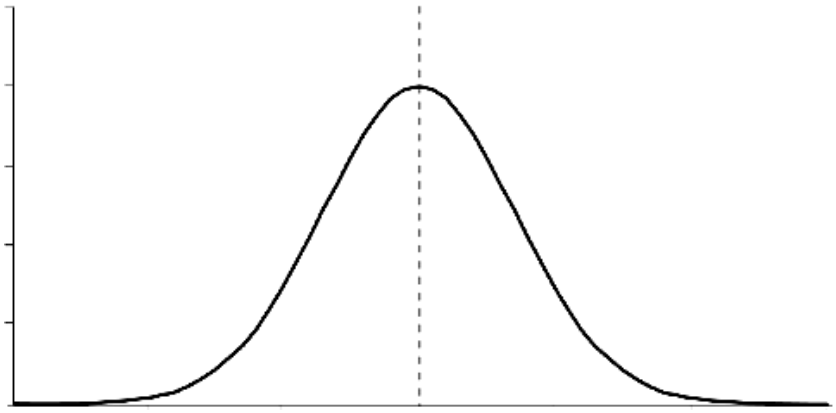
MOD 13 PART 1 – AREA UNDER THE NORMAL DENSITY CURVE

Learning Goals

- Graph a normal density curve and summarize its important properties
- Apply the empirical rule (the 68-95-99.7 rule) to solve simple probability problems

1) The scores on a statistics exam were approximately normal with $\mu = 78$ and $\sigma = 7$. Complete the density curve for the normal distribution of exam scores. Mark and label the location of the mean and one, two, and three standard deviations above and below the mean.

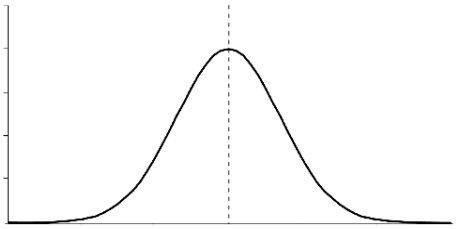


a) Now mark the middle 68%, 95% and 99.7% of the graph. And then state the ***Empirical Rule*** (a.k.a. the 68-95-99.7 rule).

b) What do you think the total area under this curve is? Explain. Hint: you may need to review number 10f from the Mod 12 worksheet.

c) What do you think the area under the curve is for the middle 68% of the data? How about the areas for the middle 95% and the middle 99.7% of the data?

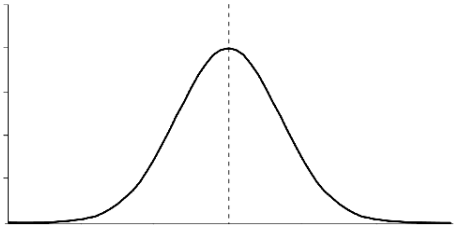
d) What is the probability that a randomly selected student scored between 71 and 85 on the statistics exam?



Which of the following statements is true about this problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

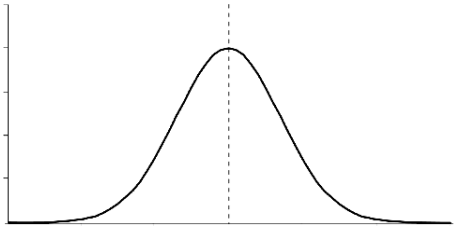
e) What was the range of scores for the lowest 16% of the class?



Which of the following statements is true about this problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

f) What is the probability that a randomly selected student scored higher than 85?

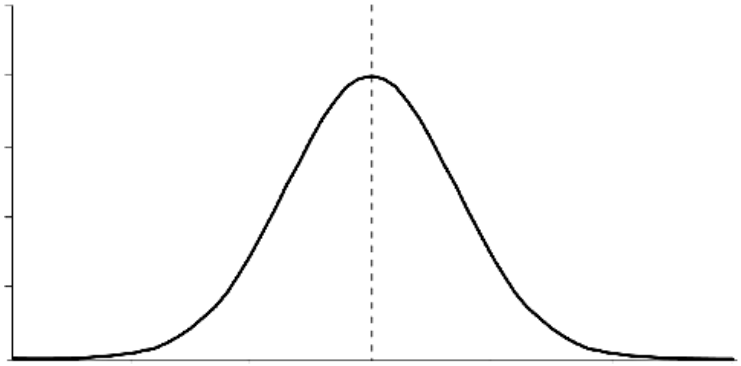


Which of the following statements is true about this problem.

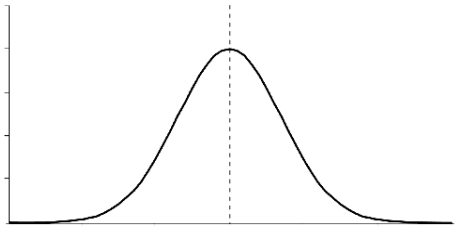
- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

2) The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with $\mu = 266$ days and $\sigma = 16$ days.

a) Complete the density curve for the normal distribution of pregnancy lengths. Mark and label the location of the mean and one, two, and three standard deviations above and below the mean.



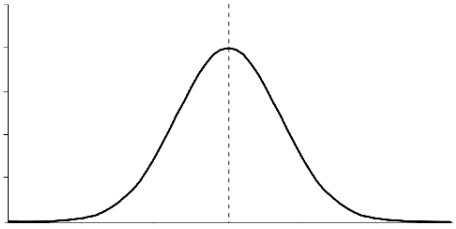
b) What are the upper and lower bounds for the middle 99.7% of all pregnancy lengths?



Which of the following statements is true about this problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

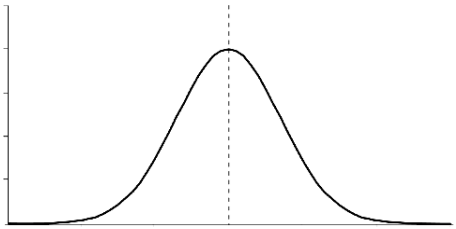
c) What is the probability that a pregnancy will last between 266 and 282 days?



Which of the following statements is true about this problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

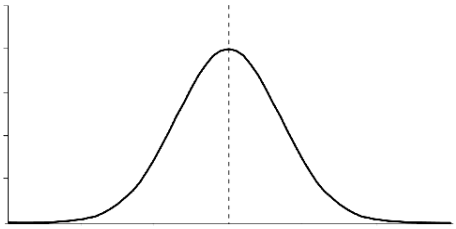
d) What is the probability that a pregnancy will last between 234 and 314 days?



Which of the following statements is true about this problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

e) How long are the longest 2.5% of pregnancies?



Which of the following statements is true about this problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

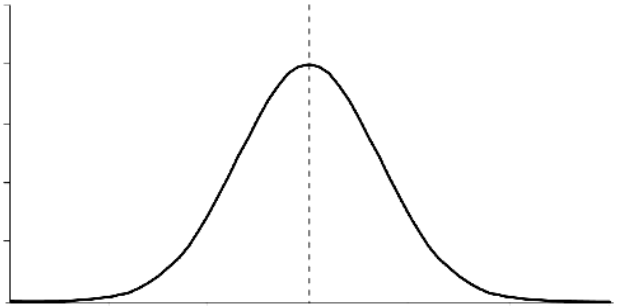
MOD 13 PART 2 INTRODUCTION TO NORMAL QUANTITATIVE VARIABLES

Learning Goals

- Use a normal probability distribution to estimate probabilities and identify unusual events

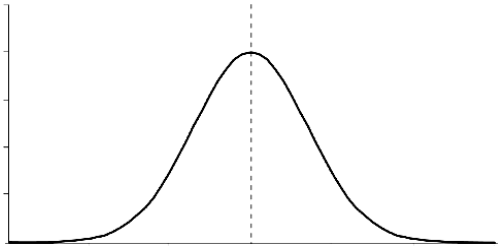
1) The height *X* (in inches) of North American women has the N(64, 2.7) distribution (What does this notation mean???).

   a) Draw the probability density curve for the normal distribution of heights of women (if you don't know what values to place on each axis or how to draw a Normal density curve, be sure to ask).



Do you think the height of a randomly selected woman is a **discrete** or **continuous** quantitative variable? Circle one.

   b) What is the probability that a randomly selected woman will be between 58.6 and 72.1 inches tall? Be sure to write your result using probability notation.

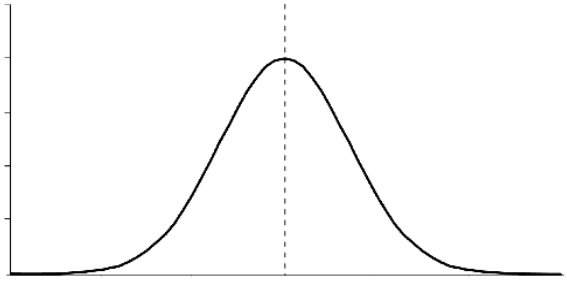


Which of the following statements is true about this problem.

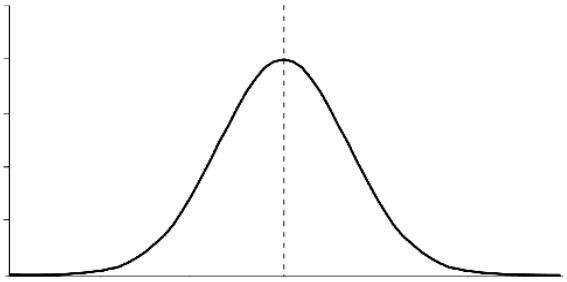
- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

c) What is the probability that a randomly selected woman will be:

shorter than 5 feet (60 inches)?



between 68 and 72 inches tall?



d) Hmmm … what are the limitations of the 68-95-99.7 rule?

2) Before we can find a probability for a value that is not 1, 2, or 3 standard deviations above or below the mean, we need to learn how to determine how many standard deviations below or above the mean that value actually is. For example we need to know how many standard deviations above or below the mean 60 inches is for the N(64, 2.7) distribution. So, we need to learn about **the standard normal distribution** and z-scores. If *X* is an observation from a distribution that has mean μ and standard deviation σ, the **standardized value** (a.k.a. the z-score) of *X* is $z = \frac{x-\mu}{\sigma}$. The z-score indicates the number of standard deviations *X* is from the mean.

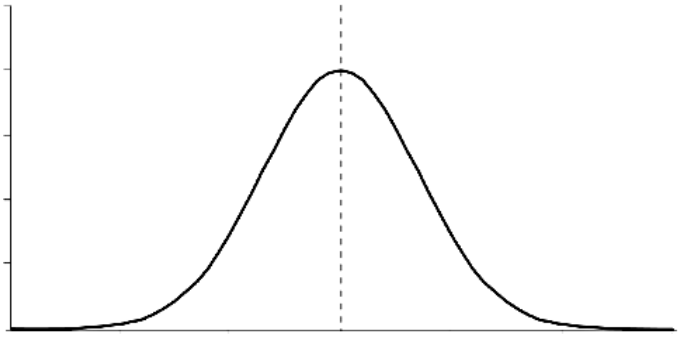
Use the z- score formula $z = \frac{x-\mu}{\sigma}$ to standardize the heights of women in the table below.

***X*: Heights of Women (inches)**	***Z*: Standardized Heights of Women**
55.9	
58.6	
61.3	
64	
66.7	
69.4	
72.1	

a) What is the standardized mean height of women, and what is the standardized standard deviation for the heights of women?

b) How many standard deviations above or below the mean is a height of 60 inches? Show your work and write your answers using complete sentences.

c) Draw the density curve for the **standard normal distribution** N(0, 1), i.e. in this case, the standardized density curve for the heights of women. Label the curve with the *X* and *Z* values from the table. And then label the curve with the *X* and *Z* values for heights from part b) above.



If we define an ***unusual value*** to be any value that is two or more standard deviations below or two or more standard deviations above the mean, is a height of 62 inches unusual for a woman? How about a height of 72 inches?

3) As previously stated the height *X* (in inches) of North American women has the N(64, 2.7) distribution. Also the height *X* (in inches) of North American men has the N(69, 2.9) distribution.

   Ms. Hilda Branstetter is 69.67 inches tall, and Mr. Borna Jabbapore is 74.22 inches tall.

   a) Relative to their respective genders, who is taller? Support your conclusion.

   b) Relative to their respective genders is either height unusual? How do you know?

4) For this part of the activity we will introduce the following applets. Note: you can access these applets in the Resources module of our course on Canvas. Note: all three applets should work on your smart phone as well.

   Inverse Normal Distribution Calculator

   Input: area; mean = 0; SD = 1; and whether you want to find the z value for an area above, below, between, or outside

   Output: the z-score(s) as boundaries for the given area

   Normal Distribution Calculator

   Input: single z-score (one boundary)

   Output: the area to the left and the area to the right of the z-score

   Z-score Calculator

   Input: two z-scores (boundaries)

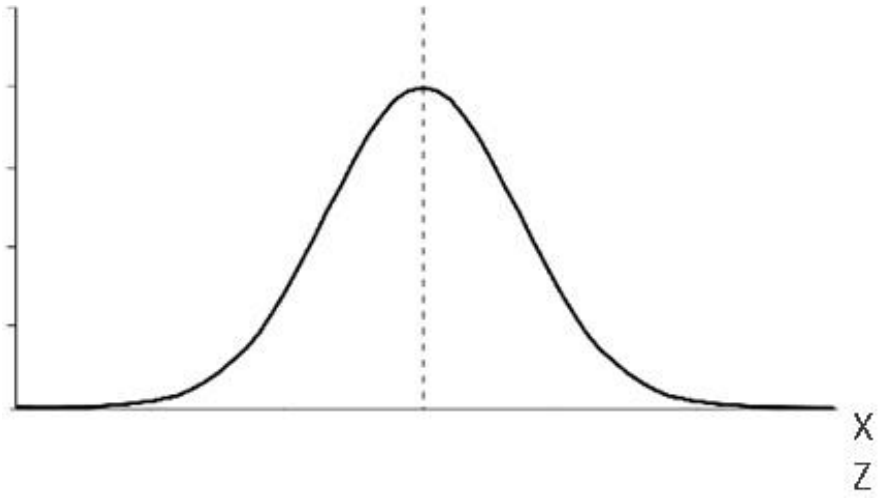
   Output: Output: area in between the z-scores

Continued on the next page …

Recall the height *X* (in inches) of North American women has the N(64, 2.7) distribution.

a) Let's find the probability that a randomly selected woman will be shorter than 60 inches (5 ft) tall.

   Step 1: Complete the probability density curve for the normal distribution of heights of women. Shade in the appropriate region (shorter than 60 inches), and be sure to label the curve with the appropriate values.



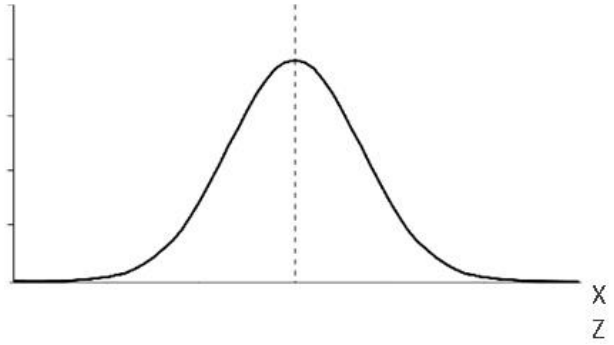
   Step 2: Determine which of the following statements is true about the problem.
   - You were given an area and asked to find the boundaries.
   - You were given the boundaries and asked to find the area.

   Step 3: Based on your response in Step 2 select and use the appropriate applet to answer the question. So which applet will you use? Explain your choice.

   What is the probability that a randomly selected woman will be shorter than 60 inches (5 ft) tall?

b) What is the probability that a randomly selected woman will be between 68 and 72 inches tall?

Step 1: Complete the normal density curve. Be sure to shade in the appropriate region and label the curve with the appropriate values.



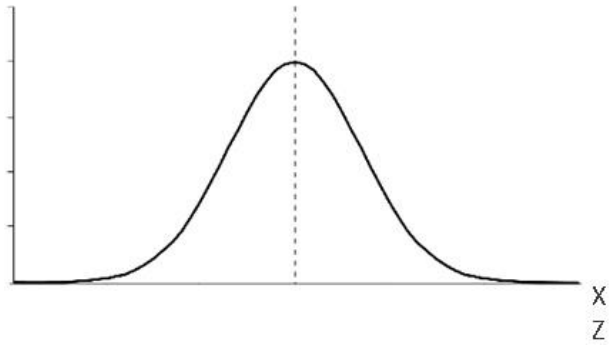
Step 2: Determine which of the following statements is true about the problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

Step 3: Based on your response in Step 2 select and use the appropriate applet to answer the question.

c) How tall is a North American woman who is in the 75th percentile?

Step 1: Complete the normal density curve. Be sure to shade in the appropriate region and label the curve with the appropriate values.



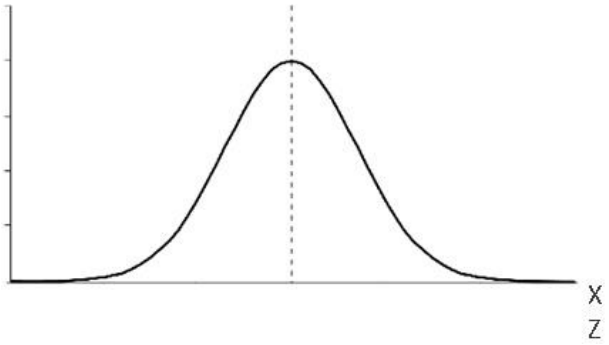
Step 2: Determine which of the following statements is true about the problem.

- You were given an area and asked to find the boundaries.
- You were given the boundaries and asked to find the area.

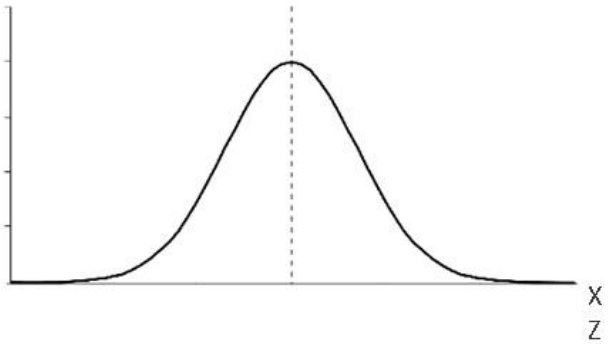
Step 3: Based on your response in Step 2 select and use the appropriate applet to answer the question.

5) An average light bulb manufactured by the Acme Corporation lasts 300 days with a standard deviation of 50 days. Assume that bulb life is normally distributed.

a) Use the steps described in number 4 above to assess the probability that a randomly selected Acme light bulb will last at most 365 days? Write the probability notation with an inequality. Also, be sure to complete all three steps and then indicate which applet you used.

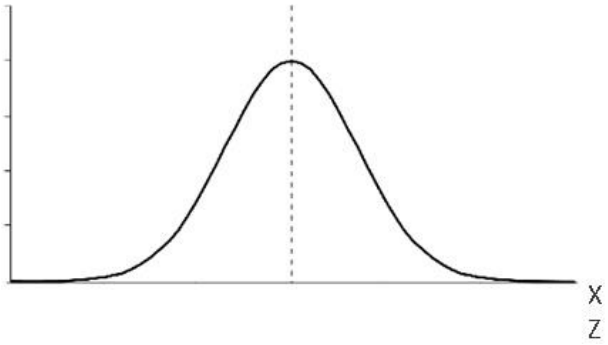


b) What is the probability that an Acme light bulb will last between 6 and 18 months (assume one month is 30 days)? Write the probability notation with an inequality. Also, be sure to complete all three steps and then indicate which applet you used.

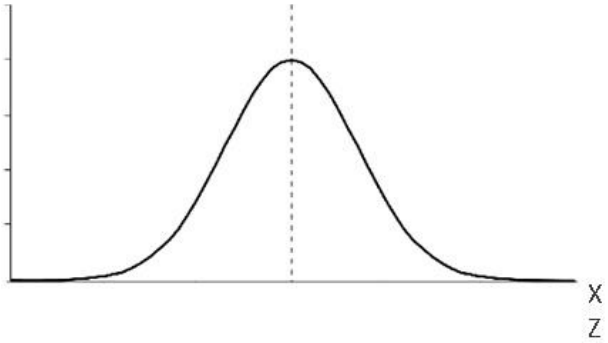


6) Suppose scores on an IQ test are normally distributed with a mean of 100 and a standard deviation of 10.

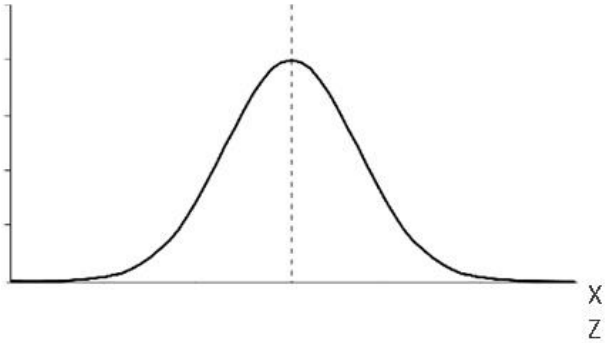
   a) Find the range of scores for the bottom 20% of students. Be sure to complete all three steps and then indicate which applet you used.



   b) Find the range of scores for the top 20% of students. Be sure to complete all three steps and then indicate which applet you used.

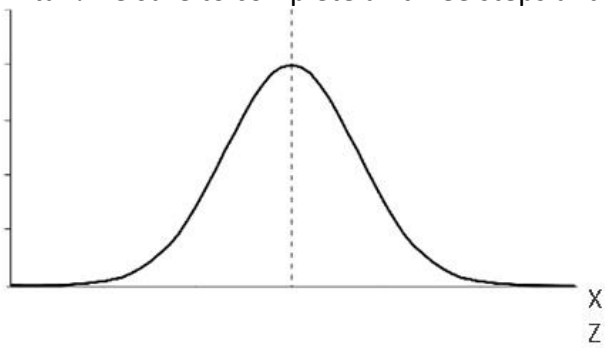


   c) Find the range of scores for the middle 70% of students. Be sure to complete all three steps and then indicate which applet you used.

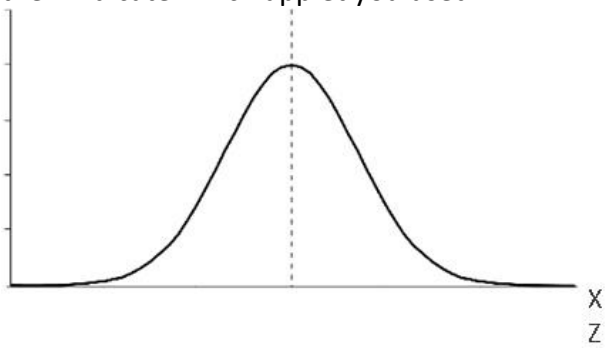


7) In a survey of men in the United States (ages 20 to 29), the mean height was 69.9 inches with a standard deviation of 3.0 inches. Assume this height data is normally distributed.

a) What is the probability that a randomly selected man is between 5 ft 9 in and 6 ft 6 in tall? Be sure to complete all three steps and then indicate which applet you used.



b) What height represents the 25th percentile? Be sure to complete all three steps and then indicate which applet you used.



c) How tall are the middle 75% of men? Be sure to complete all three steps and then indicate which applet you used.