MOD 17 ESTIMATING A POPULATION PROPORTION

Learning Goals

- Find a confidence interval to estimate a population proportion when conditions are met, and interpret the confidence interval in context
- Determine whether conditions are met for estimating the population proportion, p
- 1) What is the purpose of a confidence interval?
- 2) Which of the following questions would be answered by a confidence interval? Explain your choice(s).
 - a) What is the average amount of money that community college students receive in financial aid?
 - b) Do the majority of community college students qualify for federal student loans?
 - c) What proportion of California Community College students qualify for a BOG waiver?
- 3) When we want to estimate a population proportion at the 95% confidence level, we use the following formula.

sample proportion \pm margin of error = $\hat{p} \pm 2\sqrt{\frac{p(1-p)}{n}}$

- a) Dissect the right side of the formula and explain what each piece represents.
- b) The standard error (SE) formula uses the population proportion, p, but we're supposed to use the formula to estimate the population proportion, so we don't know the value of p. So how do we handle this?
- c) The margin of error is based on the standard error. What does the standard error tell us?
- d) At the 95% confidence level, why is the margin of error 2 times the standard error as opposed to 1.5 times or 3 times the standard error?

- e) What conditions must be met before we can use this formula to find a confidence interval?
- 4) As discovered above, the equation for a confidence interval for a proportion has been given as $\hat{p} \pm MOE$. Now let's review the formula for calculating the MOE.

margin of error = (critical value)(standard error)

At a given confidence level the *critical value* indicates the number of standard deviations we move away from the mean to arrive at the lower and upper boundaries of a confidence interval. So the *critical value* is a Z-score, and is denoted Z_c .

- a) What is the critical value at the 95% confidence level?
- b) What is the critical value at the 99.7% confidence level?
- c) Convert the formula *margin of error* = (*critical value*)(*standard error*) to symbols.
- 5) Wait we'll work on this one as a class. Your answers to parts a) and b) in the previous problem were likely based on the Empirical rule (the 68-95-99.7 rule). We will now use an applet or a calculator to find a more accurate critical value.



Find the critical values for some common confidence intervals:

- 90% critical value = $Z_{0.90}$ =
- 95% critical value = $Z_{0.95}$ =
- 99% critical value = $Z_{0.99}$ =

6) Hershey's Kisses and Confidence Intervals

In this activity, we will estimate a confidence interval for the proportion of times a Hershey's Kiss lands on its base as opposed to its side. To do this, we will drop Hershey's Kisses, count how many land on their base, and calculate the confidence interval.

Gather five Hershey's Kisses in your cup, shake them up, and drop them from about six inches above your desk. Count the number that land on their base. Repeat the 10 times to get a sample of 50 combined tosses.

Record your results in the table below.



" I got the instructions from my Statistics Professor. He was 80% confident that the true location of the restaurant was in this neighborhood."

Toss	Number that
Number	land on base
1	
2	
3	
4	
5	

Toss	Number that	
Number	land on base	
6		
7		
8		
9		
10		

a) Out of 50 tossed Hershey's Kisses, how many landed on the base?

- b) What is the population of interest?
- c) What is the sample?
- d) Proportion of Hershey's Kisses (50 tosses combined) that landed on the base: \hat{p} =
- e) Compare your \hat{p} with the other groups. Did you all get the same answer?

We're still working on the Hershey's Kisses activity.

- f) Determine whether the conditions are met for constructing a confidence interval using your sample of 50 tossed kisses.
- g) If possible, construct the 90% confidence interval based on your results. Write the lower bound and upper bound in the spaces below. Show your work.

lower bound = _____

upper bound = _____

- h) When finished mark your interval and \hat{p} on the board that your instructor set up. After reviewing all the confidence intervals on the board, what can we conclude?
- i) Interpret the 90% confidence interval in your own words. Write your interpretation below.
- j) If you've completed these tasks and questions above, eat a Hershey's Kiss while you compare your interpretation with your group members' interpretations and come to agreement on the appropriate interpretation. Write it below.

Pulling it All Together: Confidence Intervals - A Four-Step Process

<u>Step 1</u>: Identify the *parameter* you want to estimate and the *confidence level*.

Step 2: Check conditions.

<u>Step 3</u>: If the conditions are met, perform the *calculations*.

<u>Step 4</u>: State the result (the confidence interval) and *interpret* your result in the *context* of the problem.

7) In October, 2018, Gallup surveyed a random sample of 1,019 U.S. adults and asked, "Generally speaking, do you believe the death penalty is applied fairly or unfairly in this country today?" Forty-nine percent of respondents indicated that the death penalty was applied fairly. Use the four-step process to estimate the proportion of U.S. adults who believe the death penalty is applied fairly. Select either a 75%, 80%, or 85% confidence level.

<u>Step 1</u>: Identify the *parameter* you want to estimate and the *confidence level*.

Step 2: Check conditions.

<u>Step 3</u>: If the conditions are met, perform the *calculations*.

<u>Step 4</u>: State the result (the confidence interval) and *interpret* your result in the *context* of the problem.

- 8) In March, 2019, Gallup published an online article by Frank Newport titled, "American's Views of Israel Remain Tied to Religious Beliefs." For the article Mr. Newport reviewed data previously collected by Gallup in several national random samples and focused on the responses from 128 Jewish Americans. Of these respondents, 110 indicated that they were sympathetic toward Israel.
 - a) Use the four-step process to construct and interpret a 95% confidence interval for the proportion Jewish Americans who are sympathetic toward Israel.

b) Find the 99% confidence interval (no need to interpret).

c) Find the 90% confidence interval (no need to interpret).

Continued on the next page ...

d) Rewrite all three confidence intervals from the previous page.

90% confidence interval:

95% confidence interval:

99% confidence interval:

As the confidence level went up, what happened to the confidence interval?

What can you say about how precision (width of the confidence interval) and confidence level are related?

How can we have *both* a narrow interval *and* a high level of confidence? Note: you may not be able to answer this question yet. But we'll do a little exploration in the next module that may be able to help.

