Mod 22 Hypothesis Test for a Population Mean

Learning Goals

- Carry out a significance test for μ
- Calculate one-sample *t* test
- Calculate two-sided tests and confidence intervals
- Use matched paired data to conduct inference for means
- Use tests wisely

Just like confidence intervals, we can use significance tests for population means.

1) **Conditions** What are the three conditions for conducting a significance test for a population mean?

We often don't know whether the population distribution is Normal. If the sample size is large ($n \geq 30$), we can safely carry out the significance test (due to the central limit theorem). If the sample size is small, we should examine the sample data for any obvious departures from Normality, such as skewness and outliers. What types of graphs can help us check for skewness and/or outliers?

2) **Less music?**

A classic rock radio station claims to play an average of 50 minutes of music every hour. However, it seems that every time you turn to this station, there is a commercial playing. To investigate their claim, you randomly select 12 different hours during the next week and record what the radio station plays in each of the 12 hours. Here are the number of minutes of music in each of these hours:

44 49 45 51 49 53 49 44 47 50 46 48

Check the conditions for carrying out a significance test of the company's claim that it plays an average of 50 minutes of music per hour.

Random:	*Normal:*	*Independent:*

Carry out the significance test. Use the significance level α = 0.05

<u>Step 1</u>: State the ***claim*** and its opposite. Identify which is the ***null hypothesis*** and which is the ***alternative hypothesis***.

<u>Step 2</u>: Determine ***which hypothesis test*** you will use and ***check conditions***.

<u>Step 3</u>: If the conditions are met, perform the ***calculations*** and ***conduct the test***.
(Hint: See Calculator Directions at the end of this activity.)

<u>Step 4</u>: State the result and ***interpret*** your result in the ***context*** of the problem.

3) **Healthy Streams**

The level of dissolved oxygen (DO) in a stream or river is an important indicator of the water's ability to support aquatic life. A researcher measures the DO level at 15 randomly chosen locations along a stream. Here are the results in milligrams per liter (mg/l):

4.53	5.04	3.29	5.23	4.13	5.50	4.83	4.40
5.42	6.38	4.01	4.66	2.87	5.73	5.55	

A dissolved oxygen level below 5 mg/l puts aquatic life at risk.

a) Can we conclude that aquatic life in this stream is at risk? Carry out a test at the α = 0.05 significance level to help you answer this question.

Step 1: State the ***claim*** and its opposite. Identify which is the ***null hypothesis*** and which is the ***alternative hypothesis***.

Step 2: Determine ***which hypothesis test*** you will use and ***check conditions***.

Step 3: If the conditions are met, perform the ***calculations*** and ***conduct the test***. (Hint: See Calculator Directions at the end of this activity.)

Step 4: State the result and ***interpret*** your result in the ***context*** of the problem.

b) Given your conclusion in part (a), which kind of mistake—a Type I error or a Type II error—could you have made? Explain what this mistake would mean in context.

Two-sided Tests

4) **Juicy Pineapple**

At the Hawaii Pineapple Company, managers are interested in the sizes of the pineapples grown in the company's fields. Last year, the mean weight of the pineapples harvested from one large field was 31 ounces. A new irrigation system was installed in this field after the growing season. Managers wonder whether this change will affect the mean weight of future pineapples grown in the field. To find out, they select and weigh a random sample of 50 pineapples from this year's crop. The Minitab output below summarizes the data.

Descriptive Statistics: Weight (oz)

Variable	N	Mean	SE Mean	St Dev	Minimum	Q1	Median	Q3	Maximum
Weight (oz)	50	31.935	0.339	2.394	26.491	29.99	31.739	34.115	35.547

a) Determine whether there are any outliers. Show your work.

b) Do these data suggest that the mean weight of pineapples produced in the field has changed this year? Give appropriate statistical evidence to support your answer.

Step 1:

Step 2:

Step 3:

Step 4:

c) Can we conclude that the new irrigation system *caused* a change in the mean weight of pineapples produced? Explain your answer.

MOD 22 MATCHED-PAIRS DESIGN

Introduction

When performing a hypothesis test with two sets of quantitative data that we wish to compare, we use a ***matched-pairs* design** for "before and after" situations. So the matched-pairs design is used when each individual in a sample has two quantitative measurements (typically, but not always, a *before* measurement and an *after* measurement). For each individual, we subtract the two quantitative measurements and use the difference as a single quantitative measurement for that individual. Using the differences obtained from the individuals in the sample, we can then proceed with our calculations as if we have one set of quantitative data.

When do we use a matched-pairs design for a situation that is not "before and after?" When we pair individuals with similar characteristics (think twins) and it is appropriate to subtract the quantitative measurements to get a single quantitative measurement (the difference) for each pair.

5) For each of the following pairs of data sets, indicate whether or not a *matched-pairs* design may be appropriate. Explain your choice.

   a) **Data set 1:** Resting heart rates of 35 individuals before drinking coffee
      Data set 2: Resting heart rates of the same individuals after drinking two cups of coffee

   b) **Data set 1:** Test scores for 35 statistics students
      Data set 2: Test scores for 42 biology students who do not study statistics

   c) **Data set 1:** Heights of 27 females
      Data set 2: Heights of 27 males

   d) **Data set 1:** Midterm exam scores of 14 chemistry students
      Data set 2: Final exam scores of the same 14 chemistry students

Hold up – We'll start number 6 together as a class.

6) **Is Caffeine Dependence Real?** Researchers designed an experiment to study the effects of caffeine withdrawals. They recruited 11 volunteers who were diagnosed as being caffeine dependent to serve as subjects. Each subject was barred from coffee, colas, and other substances with caffeine for the duration of the experiment. During on two-day period, subjects took capsules containing their normal caffeine intake. During another two-day period, they took placebo capsules. The order in which subjects took caffeine and the placebo was randomized. At the end of each two-day period, a test for depression was given to all 11 subjects. Researchers wanted to know whether being deprived of caffeine would lead to an increase in symptoms of depression.

The table below contains data on the subject's scores on a depression test. Higher scores show more symptoms of depression.

Results of a caffeine-deprivation study

Subject	Depression (caffeine)	Depression (placebo)
1	5	16
2	5	23
3	4	5
4	3	7
5	8	14
6	5	24
7	0	6
8	0	3
9	2	15
10	11	12
11	1	0

a) Why did researchers randomly assign the order in which subjects received placebo and caffeine?

b) Carry out a test to investigate the researchers' question.

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Step 2: Determine ***which hypothesis test*** you will use and ***check conditions***.

Step 3: If the conditions are met, perform the ***calculations*** and ***conduct the test***.

Step 4: State the result and ***interpret*** your result in the ***context*** of the problem.

7) **Is the express lane faster?**

For their statistics project Libby and Kathryn decided to investigate which line was faster in the supermarket: the express lane or the regular lane. To collect their data, they randomly selected 15 times during a week, went to the same store, and bought the same item. However, one of them used the express lane and the other used a regular lane. To decide which lane each of them would use, they flipped a coin. If it was heads, Libby used the express lane and Kathryn used the regular lane. If it was tails, Libby used the regular lane and Kathryn used the express lane. They entered their randomly assigned lanes at the same time, and each recorded the time in seconds it took them to complete the transaction.

Time in regular lane (seconds)	**Time in express lane (seconds)**
342	337
472	226
456	502
529	408
181	151
339	284
229	150
263	357
332	349
352	257
341	321
397	383
694	565
324	363
127	85

Carry out a test to see if there is convincing evidence that the express lane is faster.

State:

Plan:

Do:

Conclude:

MOD 22 GRAPHING CALCULATOR DIRECTIONS

Checking Conditions Histograms, Boxplots and Normal probability plots

1. Enter the data: press *stat* and choose *1: Edit*. Type values into list L1.
2. Press *2nd Y =* to get into STAT PLOT. Press *Enter* or *1* to get into Plot1. Set the plot to *on*.
3. Choose the type you want histogram (the third picture) or boxplot (the fourth picture) or normal probability plot (the sixth picture)
4. Verify XList: L_1 (or whichever area you entered your data)
5. Press *zoom* and then *9:zoomstat*
6. If an adjustment is needed on the classes of the histogram, Do the following:
    a) Press *window* and enter your new values
    b) Press *graph*
    c) Press *trace* and arrows left and right to examine the new values.