MOD 23 – INFERENCE FOR A DIFFERENCE BETWEEN POPULATION MEANS

Learning Goals

- Perform a significance test to compare two means
- Check conditions for using two-sample t procedures in a randomized experiment
- Interpret the results of inference procedures in a randomized experiment
- Determine the proper inference procedure to use in a given setting

Caffeine and Pulse Rate

In a statistics class an experiment was conducted where 20 subjects received exactly the same amount of cola served at the same temperature. Also, each type of cola looked the same, tasted the same and had the same amount of sugar. Subjects drank the cola at the same rate and waited the same amount of time before measuring their pulse rates. Each subject was given randomly either cola with caffeine or cola without caffeine. Each subject recorded their initial pulse rates, drank the soda and recorded their final pulse rates.

The increase in pulse rates for each student is shown below (a negative value indicates a decrease in pulse rate.)

Caffeine:	14	3	-1	0	0	3	4	-1	2	0	5	1
Caffeine Free:	6	0	2	1	-2	-3	1	4				

Find the mean for the caffeine group $\bar{x}_c =$

Find the mean for the caffeine-free $\bar{x}_{cf} =$

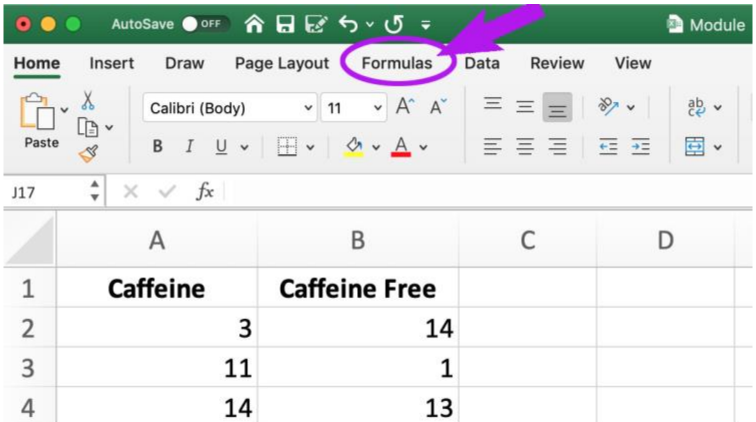
What is the difference between the mean of the caffeine group and the mean of the caffeine-free group? Round to one decimal place.

$\bar{x}_c - \bar{x}_{cf} =$

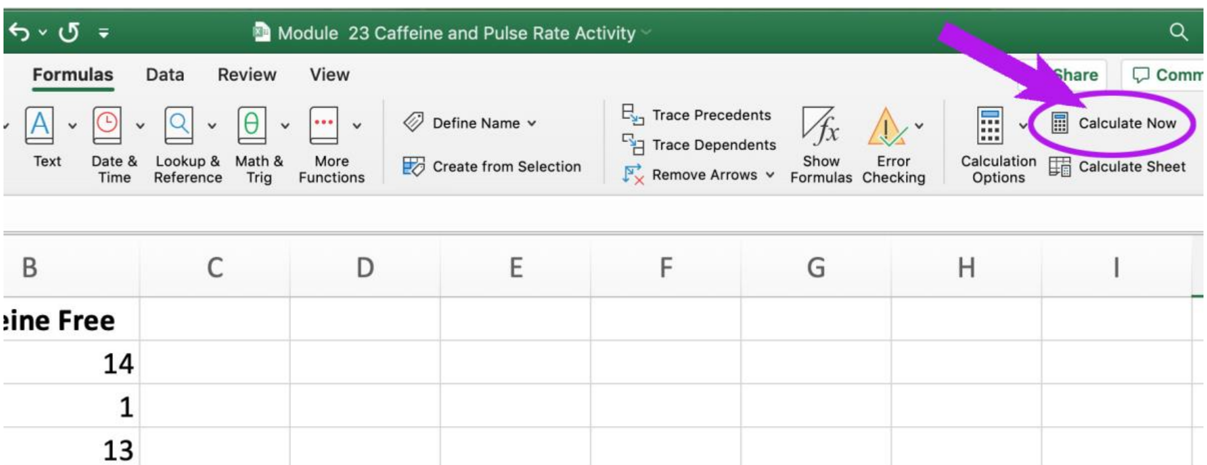
Does this result provide convincing evidence that caffeine increases pulse rates? Why or why not?

Let's try a simulation. Your instructor will provide access to the ***Module 23 Caffeine and Pulse Rate Activity*** Excel file.

To create a new random sample for each group, select the ***Formulas*** tab.



Then click on ***Calculate Now***.



Generate 10 sets of random samples and record your results in the table below.
Round $\bar{x}_c - \bar{x}_{cf}$ to one decimal place. Then continue on to the next page.

$\bar{x}_c$										
$\bar{x}_{cf}$										
$\bar{x}_c - \bar{x}_{cf}$										

When your group has completed the table, someone from the group will need to plot the 10 points for the difference of the means ($\bar{x}_c - \bar{x}_{cf}$) on the dotplot at the front of the room. Use a blue marker to plot the value for data points less than 1.4. Use a red marker to plot the values for data points that are 1.4 or greater. After each group has graphed their data, answer the questions below.

1) How many dots are there on 1.4 and above?

2) How many total dots are there in the graph?

3) If we randomly select a difference of means, what is the probability it is 1.4 or greater?

   (amount of dots above 1.4)/(total number of dots) =

4) Is the p-value calculated in number 3 significant?

Confidence Intervals for the difference of means μ_1- μ_2

If the random and normal conditions are met, we can use our standard formula to construct a confidence interval:

CI = Statistic ± (critical value)(standard deviation of statistic)

- **Random** the data are produced by a random sample of size n_1 from population 1 and a random sample of size n_2 from population 2 or two groups of size n_1 and n_2 in a randomized experiment.
- **Normal** Both population distributions (or the true distributions of responses to the two treatments) are Normal OR both sample/group sizes are large ($n_1 \geq 30$ and $n_2 \geq 30$)

Confidence Intervals Example 1: *Big Trees, Small Trees, Short Trees, Tall Trees*

The Wade Tract Preserve in Georgia is an old-growth forest of longleaf pines that has survived in a relatively undisturbed state for hundreds of years. One question of interest to foresters who study the area is "How do the sizes of longleaf pine trees in the northern and southern halves of the forest compare?" To find out researches took random samples of 30 trees from each half and measured the diameter at breast height (DBH) in centimeters. For the north side the mean was 23.70 cm with a standard deviation of 17.50 and for the south side the mean was 34.53 with a standard deviation of 14.26. Construct and interpret a 90% confidence interval for the difference in the mean DBH of longleaf pines in the northern and southern halves of the Wade Tract Preserve.

<u>Step 1</u>: Identify the ***parameter*** you want to estimate and the ***confidence level***.

<u>Step 2</u>: Check ***conditions***.

<u>Step 3</u>: If the conditions are met, perform the ***calculations***.

<u>Step 4</u>: State the result (the confidence interval) and ***interpret*** your result in the ***context*** of the problem.

Confidence Intervals Example 2: *Plastic Grocery Bags*

Do plastic bags from Target or plastic bags from Bashas hold more weight? A group of statistics students decide to investigate by filling a random sample of 5 bags from each store with common grocery items until the bags ripped. Then they weighed the contents of items in each bag to determine its capacity. Here are their results, in grams:

Target	12,572	13,999	11,215	15,447	10896
Bashas	9552	10896	6983	8767	9972

Construct and interpret a 99% confidence interval for the difference in mean capacity of plastic grocery bags from Target and Bashas?

Step 1: Identify the ***parameter*** you want to estimate and the ***confidence level***.

Step 2: Check ***conditions***.

Step 3: If the conditions are met, perform the ***calculations***.

Step 4: State the result (the confidence interval) and ***interpret*** your result in the ***context*** of the problem.

What happens if we change our confidence level to 95%? Does our conclusion change and why?

Significant Tests for μ_1 - μ_2

If conditions are met, we can proceed with a two-sample t-test. The conditions are the same as the confidence interval. Complete the formula below for the test statistic.

Test statistic = $\frac{statistic - parameter}{standard\ deviation\ of\ statistic}$ =

Significance Test Example 1: *Calcium and Blood Pressure*

Does increasing the amount of calcium in our diet reduce blood pressure? Examination of a large sample of people revealed a relationship between calcium intake and blood pressure. The relationship was strongest for black men. Such observational studies do not establish causation. Researchers therefore designed a randomized comparative experiment.

The subjects were 12 healthy black men who volunteered to take part in the experiment. They were randomly assigned to two groups: 10 of the men received a calcium supplement for 12 weeks, while the control group of 11 men received a placebo pill that looked identical. The experiment was double-blind. The response variable is the decrease in systolic (top numer) blood pressure foa subject after 12 weeks, in millimeters of mercury. An increase appears as a negative response here are the data:

Group 1 (calcium):	7	-4	18	17	-3	-5	1	10	11	-2	
Group 2 (placebo);	-1	12	-1	-3	3	-5	5	2	-11	-1	-3

Do the data provide sufficient evidence to conclude that a calcium supplement reduces blood pressure more than a placebo? Carry out an appropriate test to support your answer.

Step 1: State the ***claim*** and its opposite. Identify which is the ***null hypothesis*** and which is the ***alternative hypothesis***.

Step 2: Determine ***which hypothesis test*** you will use and ***check conditions***.

Step 3: If the conditions are met, perform the ***calculations*** and ***conduct the test***.

Step 4: State the result and ***interpret*** your result in the ***context*** of the problem.

Significance Test Example 2: *The quicker picker-upper?*

In commercials for Bounty paper towels, the manufacturer claims that they are the "quicker picker-upper". But are they also the stronger picker upper? Two statistic students decided to find out. They selected a random sample of 30 Bounty paper towels and a random sample of 30 generic paper towels and measured their strength when wet. To do this, they uniformly soaked each paper towel with 4 ounces of water, held two opposite edges of the paper towel, and counted how many quarters each paper towel could hold until ripping, alternating brands. Here are their results:

Bounty:	106	111	106	120	103	112	115	125
	116	120	126	125	116	117	114	118
	126	120	115	116	121	113	111	128
	124	125	127	123	115	114		
Generic:	77	103	89	79	88	86	100	90
	81	84	84	96	87	79	90	86
	88	81	91	94	90	89	85	83
	84	90	100	94	87	89		

Use a significant test to determine whether there is convincing evidence that wet Bounty paper towels can hold more weight, on average than wet generic paper towels can?

Step 1: State the ***claim*** and its opposite. Identify which is the ***null hypothesis*** and which is the ***alternative hypothesis***.

Step 2: Determine ***which hypothesis test*** you will use and ***check conditions***.

Step 3: If the conditions are met, perform the ***calculations*** and ***conduct the test***.

Step 4: State the result and ***interpret*** your result in the ***context*** of the problem.