Algebra Resources

Arithmetic Operations

1. $a(b+c) = ab + ac$

2. $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad+bc}{bd}$

3. $\dfrac{a+c}{b} = \dfrac{a}{b} + \dfrac{c}{b}$

4. $\dfrac{\frac{a}{b}}{\frac{c}{d}} = \dfrac{a}{b} \times \dfrac{d}{c} = \dfrac{ad}{bc}$

Exponents and Radicals

5. $x^m x^n = x^{mn}$

6. $\dfrac{x^m}{x^n} = x^{m-n}$

7. $\left(x^m\right)^n = x^{mn}$

8. $x^{-n} = \dfrac{1}{x^n}$ **(Moving bases from the numerator to the denominator and vice versa causes the exponent to change sign.)**

9. $(xy)^n = x^n y^n$

10. $\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

11. $x^{\frac{1}{n}} = \sqrt[n]{x}$

12. $x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$

13. $\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y}$

14. $\sqrt[n]{\dfrac{x}{y}} = \dfrac{\sqrt[n]{x}}{\sqrt[n]{y}}$

Factoring Special Polynomials

15. $x^2 + y^2 = \text{prime}$

16. $x^2 - y^2 = (x+y)(x-y)$

17. $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$

Binomial Theorem

18. $(x \pm y)^2 = x^2 \pm 2xy + y^2$

19. $(x \pm y)^3 = x^3 \pm 2x^2 y + 3xy^2 \pm y^3$

20. $(x+y)^n = x^n + nx^{n-1}y + \dfrac{n(n-1)}{2}x^{n-2}y^2 + \cdots + \dbinom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$ where

$$\dbinom{n}{k} = \dfrac{n(n-1)\cdots(n-k+1)}{k!}.$$

Quadratic Formula

21. If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Inequalities and Absolute Value

22. If $a < b$ and $b < c$, then $a < c$.
23. If $a < b$, then $a + c < b + c$.
24. If $a < b$ and $c > 0$, then $ac < cb$.
25. If $a < b$ and $c < 0$, then $ac > cb$. **(Multiplying and dividing both sides of the inequality by a negative value changes the direction of the inequality.)**
26. If $a > 0$, then

   - $|x| = a$ means $x = a$ or $x = -a$.

   - $|x| < a$ means $-a < x < a$ similarly $x \in (-a, a)$.

   - $|x| > a$ means $x > a$ or $x < -a$ similarly $x \in (-\infty, -a) \cup (a, \infty)$.

27. If $a < 0$, then

   - $|x| = a$ has no solution.

   - $|x| < a$ has no solution.

   - $|x| > a$ means $x \in \mathbb{R}$.

Finding the Inverse of a Function Algebraically

28. The **Inverse Function Algorithm** starts with a one-to-one function $f(x)$.

   - **STEP 1:** Replace the function notation $f(x)$ with y so the equation gives $f(x)$ in terms of x.

   - **STEP 2:** Solve the equation for x.

   - **STEP 3:** If f is a decontextualized (abstract) function, you can interchange x and y, then replace y with $f^{-1}(x)$.

   NOTE: If f is decontextualized (abstract - no units associated with the variables), many students prefer to swap x and y first (before STEP 2), solve for y, and then replace y with $f^{-1}(x)$.