Trigonometry Resources: Angle and Function Basics

Angle Measurements

π radians $= 180^\circ$

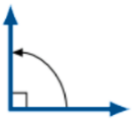
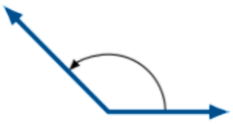
$1^\circ = \frac{\pi}{180}\text{rad}$

$1\text{ rad} = \frac{180^\circ}{\pi}$

Angle Categories

Categories by Angle Measurement

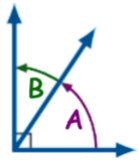
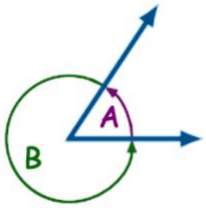
Table 1: By Measurement

Acute Angle	Right Angle	Obtuse Angle	Straight Angle
			
An angle is acute if it measures between 0° and 90°.	An angle is a right angle if it measures 90°.	An angle is obtuse if it measures more than 90°.	An angle is a straight angle if it measures 180°.

Categories by Pairwise Relationship

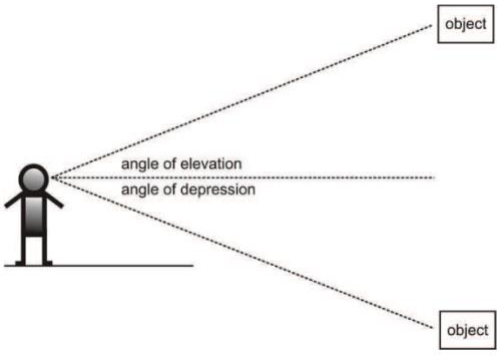
Assume the angles A and B are positive and not larger than 360°.

Table 2: By Pairwise Relationship

Complementary	Supplementary	Conjugate
		
Two acute angles are complementary if their sum is 90°, $A + B = 90^\circ$.	Two angles are supplementary if their sum is 180°, $A + B = 180^\circ$.	Two angles are conjugate if their sum is 360°, $A + B = 360^\circ$.

Angles of Elevation or Depression

The line of sight establishes a horizontal reference as the initial side of an angle. If the angle's terminal side is above the line of sight (horizon), it is an ***angle of elevation***. If the angle's terminal side is below the line of sight (horizon), it is an ***angle of depression***. These types of angles are illustrated in the diagram to the right.

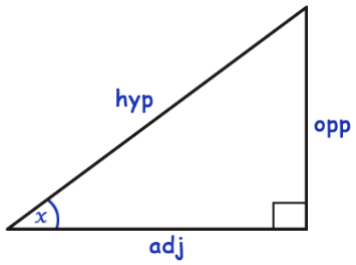


Right Angle Trigonometry

Given a right triangle, the side opposite the right angle is called the ***hypotenuse*** (hyp) and the other two sides are the ***legs*** of the triangle as shown in the diagram below. From the perspective of angle θ, opp is the length of the ***opposite leg*** and adj is the length of the ***adjacent leg***.

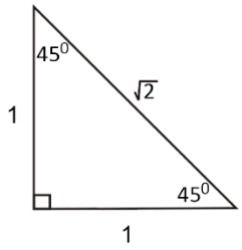
Since the angles in a triangle sum to 180° and the right angle is 90°, the other two angles must sum to 90° and each be less than 90°. So the right angle is the largest angle and the hypotenuse is the longest side of a right triangle.

Table 3: The Diagram and the Ratio of Triangles Sides By Trigonometric Function Type

Diagram	Basic Trig Functions	Reciprocal Trig Functions
	$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ $\cos\theta = \frac{\text{adj}}{\text{hyp}}$ $\tan\theta = \frac{\text{opp}}{\text{adj}}$	$\csc\theta = \frac{\text{hyp}}{\text{opp}}$ $\sec\theta = \frac{\text{hyp}}{\text{adj}}$ $\cot\theta = \frac{\text{adj}}{\text{opp}}$

Reference Triangles

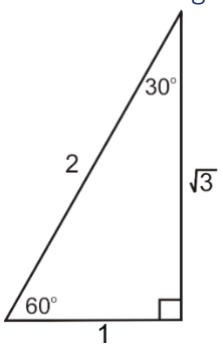
45-45-90 Triangle



When the 45-45-90 triangle is derived from a square with sides of length 1, this isosceles right triangle is obtained. Note that the length of the hypotenuse is $\sqrt{2}$ and the length of each leg is 1.

All other 45-45-90 triangles are scalar multiples of the standard 45-45-90 triangle. In other words, multiply each side of the standard 45-45-90 triangle by the same number to obtain any other 45-45-90 triangle. For example, a triangle with leg lengths of 5 and a hypotenuse of $5\sqrt{2}$ is a 45-45-90 triangle since it multiplies each side of the standard 45-45-90 triangle by 5.

30-60-90 Triangle



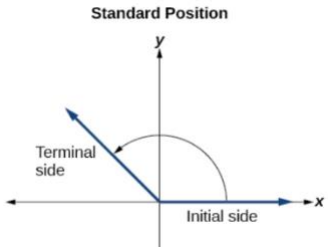
When the 30-60-90 triangle is derived from an equilateral triangle with sides of length 2, this triangle is obtained. Note that the length of the hypotenuse is 2, the length of the leg opposite the 30° angle is 1, and the length of the leg opposite the 60° is $\sqrt{3}$.

All other 30-60-90 triangles are scalar multiples of the standard 30-60-90 triangle. In other words, multiply each side of the standard 30-60-90 triangle by the same number to obtain any other 30-60-90 triangle. For example, a triangle with leg lengths of 2 and $2\sqrt{3}$ and a hypotenuse of 4 is a 30-60-90 triangle since it multiplies each side of the standard 30-60-90 triangle by 2.

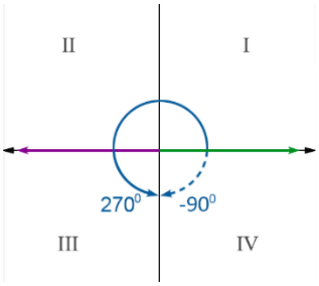
Angles in the Coordinate Plane

Standard Position

When using the rectangular coordinate system, the initial side of an angle is oriented along the positive side of the x-axis. ***Positive angles*** are measured with a ***counterclockwise*** rotation to the terminal side whereas negative angles are measured with a clockwise rotation to the terminal side.



Coterminal Angles

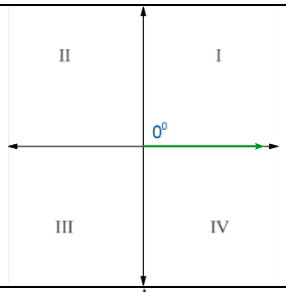
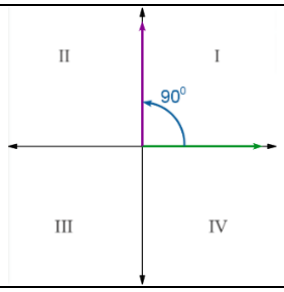
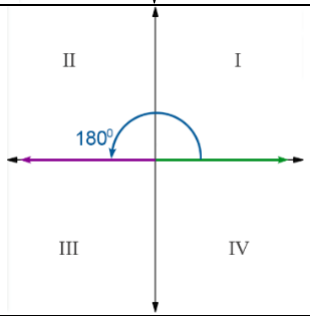
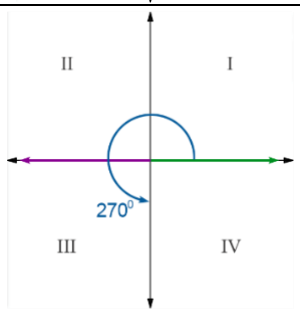
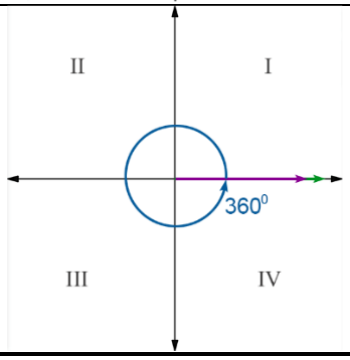


Two angles in standard position that share the same terminal side are said to be coterminal angles. The graph to the left demonstrates how 270° and -90° are coterminal angles.

Quadrantal Angles

A ***quadrantal angle*** is an angle in standard position whose ***terminal side lies along one of the axes***. So as the graphs below demonstrate, 0°, 90°, 180°, 270°, and 360° are quadrantal angles.

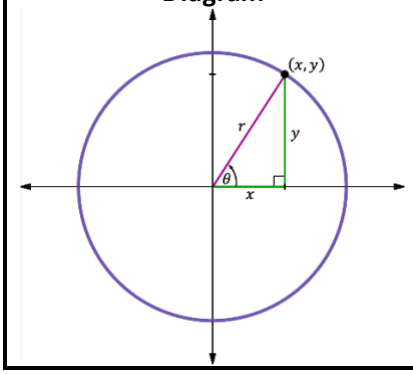
Table 4: Standard Position in the Coordinate Plane

Quadrantal Angles	
	
	
	

Trigonometric Functions for an Angle of Rotation

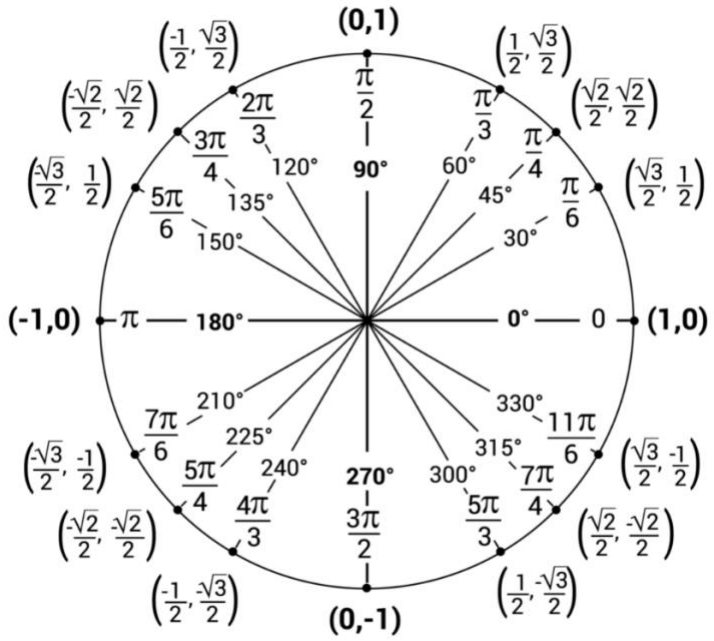
Let θ be an angle in standard position whose terminal side intersects a circle of radius r at the point (x, y) as shown in the diagram below. Using a right triangle inscribed in the circle as illustrated in the diagram, we define the six trigonometric functions at θ in terms of x, y, and r.

Table 5: The Diagram and the Trigonometric Functions for an Angle of Rotation by Trigonometric Function Type

Diagram	Basic Trig Functions	Reciprocal Trig Functions
	$\sin\theta = \frac{y}{r}$	$\csc\theta = \frac{r}{y}$
	$\cos\theta = \frac{x}{r}$	$\sec\theta = \frac{r}{x}$
	$\tan\theta = \frac{y}{x}$	$\cot\theta = \frac{x}{y}$

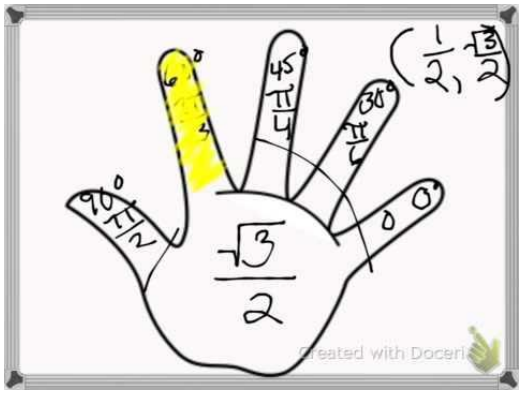
The Unit Circle

Every terminal point (x, y) represents $(\cos\theta, \sin\theta)$ on a unit circle, a circle of radius 1. The terminal points defined below are for ***critical angles***, angles that are multiples of 30° and 45° (corresponding radian measures are included).



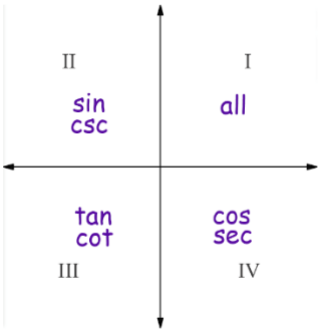
The Left-Hand Finger Trick for "Memorizing" the Unit Circle

Do you find the unit circle a little overwhelming? In the following video, ItsMsPruitt shares a left-hand five-finger trick for memorizing the first quadrant of the unit circle. The video does not lend itself to notetaking, but if you are interested work along with the video to learn the trick.



Quadrants Trigonometric Functions are Positive

All Students Take Calculus (ASTC) is one of many mnemonics for determining which of the trigonometric functions are positive in each quadrant of the plane. The letters ***ASTC*** indicate which of the trig functions are positive starting in the top right first quadrant and moving counterclockwise through the bottom right fourth quadrant.



All. In Quadrant I, ***A*** indicates all six trig functions have positive output.

Students. In Quadrant II, ***S*** indicates sine and its reciprocal function have positive output.

Take. In Quadrant III, ***T*** indicates tangent and its reciprocal function have positive output.

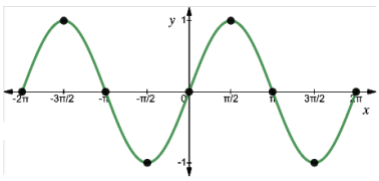
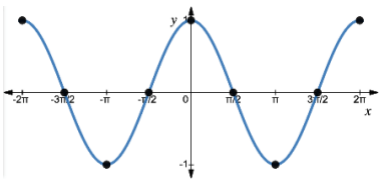
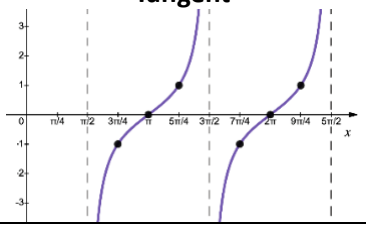
Calculus. In Quadrant IV, ***C*** indicates cosine and its reciprocal function have positive output.

Graphs of Trigonometric Functions in the Coordinate Plane

Below is the basic graph of each trigonometric function with corresponding characteristics organized by function type. For more information regarding the properties of odd or even functions, refer to the ***Negative Angle (Even and Odd)*** Identities section of the trigonometric resources page covering identities and formulas.

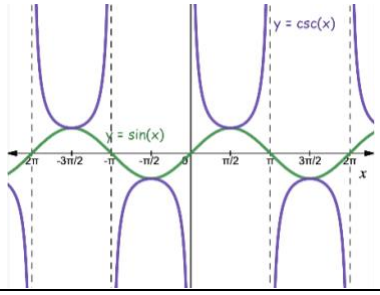
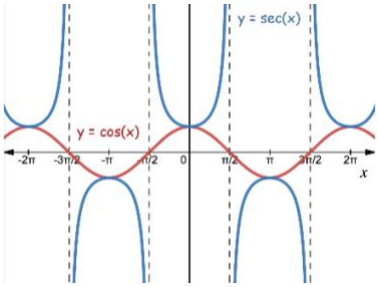
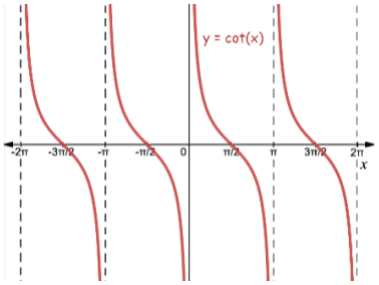
Note - the pattern established in each graph continues indefinitely to the left and to the right.

Table 6: Graph and Characteristics of Each Basic Trig Function

Sine	**Cosine**	**Tangent**
		
Domain: $(-\infty,\infty)$	**Domain:** $(-\infty,\infty)$	**Domain:** $\left\{x\in\mathbb{R}\,\middle|\,x\neq\frac{\pi}{2}+k\pi\right\}$
Range: $[-1,1]$	**Range:** $[-1,1]$	**Range:** $(-\infty,\infty)$
Period: 2π	**Period:** 2π	**Period:** π
Function Type: Odd	**Function Type:** Even	**Function Type:** Odd

Note - the pattern established in each graph continues indefinitely to the left and to the right.

Table 7: Graph and Characteristics of Each Reciprocal Trig Function

Cosecant	**Secant**	**Cotangent**
		
Domain: $\{x\in\mathbb{R}\,\vert\,x\neq k\pi\}$	**Domain:** $\left\{x\in\mathbb{R}\,\middle|\,x\neq\frac{\pi}{2}+k\pi\right\}$	**Domain:** $\{x\in\mathbb{R}\,\vert\,x\neq k\pi\}$
Range: $(-\infty,-1]\cup[1,\infty)$	**Range:** $(-\infty,-1]\cup[1,\infty)$	**Range:** $(-\infty,\infty)$
Period: 2π	**Period:** 2π	**Period:** π
Function Type: Odd	**Function Type:** Even	**Function Type:** Odd