Trigonometry Resources: Identities and Formulas

A NOTE ABOUT NOTATION:

In mathematics, there are common shorthand notations to reduce the number of parentheses used. Below are a few examples of these shorthands and their corresponding mathematical interpretations to keep in mind.

Table 1: Shorthand Notations and Interpretations

Shorthand Notation	**= Mathematical Interpretation**
$\sin\theta$	$=\sin(\theta)$
$\cos 2x$	$=\cos(2x)$
$\sec^2\theta$	$=(\sec\theta)^2=\sec\theta\cdot\sec\theta=\sec\theta\sec\theta$

Reciprocal Identities

As long as $a\neq 0$ and $b\neq 0$, the fractions $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals since $\frac{a}{b}\cdot\frac{b}{a}=1$. This means we find the reciprocal of a fraction by interchanging the numerator and the denominator, i.e. by flipping the fraction.

Table 2: Reciprocal Identities for each Trigonometric Function by Function Type

Basic Trig Function	**Reciprocal Trig Functions**
$\sin\theta=\dfrac{1}{\csc\theta}$	$\csc\theta=\dfrac{1}{\sin\theta}$
$\cos\theta=\dfrac{1}{\sec\theta}$	$\sec\theta=\dfrac{1}{\cos\theta}$
$\tan\theta=\dfrac{1}{\cot\theta}$	$\cot\theta=\dfrac{1}{\tan\theta}$

Quotient Identities

$$\tan\theta=\frac{\sin\theta}{\cos\theta}$$

$$\cot\theta=\frac{\cos\theta}{\sin\theta}$$

Cofunction Identities

Recall that ***complementary angles*** sum to 90°, or $\frac{\pi}{2}$ radians. Each ***cofunction*** has the same value at complementary angles θ and $\frac{\pi}{2}-\theta$ since $\theta+\left(\frac{\pi}{2}-\theta\right)=\frac{\pi}{2}$. In each relationship stated below, $\frac{\pi}{2}$ can be replaced with 90°.

Table 3: Cofunction Identities Grouped by Cofunction

Cofunctions		
$\sin\theta=\cos\left(\frac{\pi}{2}-\theta\right)$ $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$	$\tan\theta=\cot\left(\frac{\pi}{2}-\theta\right)$ $\cot\theta=\tan\left(\frac{\pi}{2}-\theta\right)$	$\sec\theta=\csc\left(\frac{\pi}{2}-\theta\right)$ $\csc\theta=\sec\left(\frac{\pi}{2}-\theta\right)$

Pythagorean Identities

Given the unit circle, $x^2+y^2=1$, where $x=\cos\theta$ and $y=\sin\theta$, we can define the three Pythagorean identities below.

$$\cos^2\theta+\sin^2\theta=1$$

$$1+\tan^2\theta=\sec^2\theta$$

$$\cot^2\theta+1=\csc^2\theta$$

Negative Angle (Even and Odd) Identities

Each negative angle identity is based on the symmetry of the graph of each trigonometric function. ***Even*** functions are symmetrical about the y-axis, like the quadratic function $f(x)=x^2$, and yields $f(-x)=f(x)$. ***Odd*** functions are symmetric about the origin, similar to the cubic function $f(x)=x^3$, and results in $f(-x)=-f(x)$. Below we define the output values for negative input angles.

Table 4: Negative Angle Identities Grouped by Function Type

Even	Odd	
$\cos(-\theta)=\cos\theta$ $\sec(-\theta)=\sec\theta$	$\sin(-\theta)=-\sin\theta$ $\csc(-\theta)=-\csc\theta$	$\tan(-\theta)=-\tan\theta$ $\cot(-\theta)=-\cot\theta$

Sum and Difference Identities

$$\cos(a\pm b)=\cos a\cos b\mp\sin a\sin b$$

$$\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$$

$$\tan(a\pm b)=\frac{\tan a\pm\tan b}{1\mp\tan a\tan b}$$, where $0<b<a<2\pi$

Double Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

Power Reduction Formulas

$$\cos^2\theta = \frac{\cos 2\theta + 1}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

Half Angle Identities

NOTE: For each half-angle identity, the sign to use depends on the quadrant the angle $\frac{\theta}{2}$ lies in.

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$$

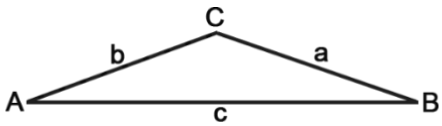
Inverse Reciprocal Formulas

Since graphing calculators only provide buttons to evaluate inverse sine, inverse cosine, and inverse tangent, use the formulas below to evaluate the inverses of the other three trig functions.

Table 5: Inverse Reciprocal Formulas by Function Type

Basic Trig Function	Reciprocal Trig Functions
$\sin^{-1}\theta = \csc^{-1}\left(\frac{1}{\theta}\right)$	$\csc^{-1}\theta = \sin^{-1}\left(\frac{1}{\theta}\right)$
$\cos^{-1}\theta = \sec^{-1}\left(\frac{1}{\theta}\right)$	$\sec^{-1}\theta = \cos^{-1}\left(\frac{1}{\theta}\right)$
$\tan^{-1}\theta = \frac{\pi}{2} - \cot^{-1}\theta$	$\cot^{-1}\theta = \frac{\pi}{2} - \tan^{-1}\theta$

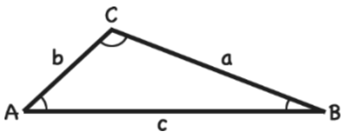
Law of Sines



Given a triangle with side a opposite angle A, side b opposite angle B, and side c opposite angle C the ***Law of Sines*** gives the following equivalences.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines



Given any triangle ABC, the ***Law of Cosines*** gives the following relationships between the sides of the triangle and one of its angles.

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$