

CUYAMACA COLLEGE COURSE OUTLINE OF RECORD

Mathematics 281 – Multivariable Calculus

4 hours lecture, 4 units

Catalog Description

The third of a three-course sequence in calculus. Topics include vector valued functions, calculus of functions of more than one variable, partial derivatives, multiple integration, Green's Theorem, Stokes' Theorem, and divergence theorem.

Prerequisite

"C" grade or higher or "Pass" in MATH C2220 (formerly MATH 280) or equivalent

Entrance Skills

Without the following skills, competencies and/or knowledge, students entering this course will be highly unlikely to succeed:

- 1) Differentiation techniques involving algebraic, trigonometric, logarithmic and exponential functions
- 2) Implicit differentiation
- 3) Integration techniques, including: substitution, integration by parts, trigonometric substitution, and trigonometric power reduction methods.
- 4) Evaluating improper integrals
- 5) Calculating limits with indeterminate forms and using L'Hospital's Rule
- 6) Graphing in rectangular and polar coordinates, using parametric equations, and conics
- 7) Modeling applications, including related rates, relative extrema, area between curves, and surface area.

Course Content

- 1) Multivariable Functions: The use of surface graphs, diagrams and tables to introduce functions of many variables. Read and interpret graphs, tables and contour maps (level curves and surfaces) in order to graphically and algebraically solve real-world problems from a variety of disciplines. Properties of limits and continuity. Finding limits of multivariable functions and determining the continuity of multivariable functions at a point.
- 2) Vectors: Vector definitions (geometric and rectangular), vector operations, dot and cross product, triple products, orthogonality, and equations of lines (vector, parametric and symmetric) and planes (rectangular and parametric).
- 3) Vector-Valued Functions: Representation of curves parametrically. Parameterized surfaces and the connection between implicit, explicit and parametric representations of surfaces. Derivatives and integrals of vector-valued functions. Application problems involving velocity and acceleration; tangent, normal, and binormal vectors; arc length and curvature.
- 4) Differentiation of Multivariable Functions: Graphic and numeric definitions and interpretation of partial derivative, directional derivative, gradient, linearization and differential. Differentiability, the multivariable chain rule and higher order partial derivatives, and their interpretation in partial differential equations.
- 5) Optimization: Both constrained and unconstrained optimization problems; the second derivative test for local extrema and saddle points; Lagrange multipliers.
- 6) Integration of Multivariable Functions: Graphical and numerical interpretation for integration of a multivariable functions. Double and triple integrals in rectangular, polar, spherical and cylindrical coordinates. Use of the Change of Variables formula. Applications of multiple integration such as area, volume, center of mass, or moments of inertia.

- 7) Vector Fields: Vector-valued functions of many variables (gradient vector fields and conservative fields) from a geometric and analytic perspective. Physical examples such as velocity fields and force fields.
- 8) Line Integrals: Line and surface integrals including parametrically defined surfaces. Integrals of real-valued functions over surfaces. Divergence and curl. Green's, Stokes' and divergence theorems.

Course Objectives

Students will be able to:

- 1) Perform vector operations:
 - a. Addition
 - b. Subtraction
 - c. Dot product
 - d. Cross product
 - e. Triple product
 - f. Orthogonality
 - g. Projection
- 2) Write equations of:
 - a. Lines
 - b. Planes
- 3) Apply the dot product, cross product, orthogonality or projection to solve problems involving:
 - a. Motion
 - b. Velocity and acceleration
 - c. Tangent, normal and binormal vectors
 - d. Arc length
 - e. Curvature
- 4) Use each of the following to analyze multivariable functions or solve real world problems:
 - a. Equations
 - b. Graphs
 - c. Contour maps
 - d. Vector fields
 - e. Tables of data
- 5) Find the limit of a function at a point.
- 6) Write the equation of a tangent plane at a point.
- 7) Test the differentiability of a variety of functions.
- 8) Evaluate partial derivatives including:
 - a. Higher order partial derivatives
 - b. Directional derivative
 - c. Gradient
 - d. Differential
- 9) Find and test for local extrema and saddle points.
- 10) Solve constraint problems using Lagrange multipliers.
- 11) Find the following using multiple integrals and sketch the region of integration:
 - a. Volume of solids
 - b. Area of plane regions
- 12) Evaluate double and triple integrals using:
 - a. Rectangular coordinates
 - b. Polar coordinates
 - c. Spherical coordinates
 - d. Cylindrical coordinates
- 13) Find the divergence and curl of a vector field.
- 14) Apply Green's, Stokes' and divergence theorems.

Method of Evaluation

A grading system will be established by the instructor and implemented uniformly. Grades will be based on demonstrated proficiency in subject matter determined by multiple measurements for evaluation, one of which must be essay exams, skills demonstration or, where appropriate, the symbol system.

- 1) Independent exploration activities, quizzes and exams which measure students' ability to apply the student learning outcomes requiring the use of multivariable calculus.
- 2) Group-work, class activities, homework and/or quizzes which measure students' ability to select and apply appropriate technology to model, analyze and interpret a collection of data or to solve real-world application problems requiring the use of multivariable calculus.

Special Materials Required of Student

Graphing utility, portfolio

Minimum Instructional Facilities

Smart classroom with writing boards covering three walls, graphing utility, overhead viewing panels

Method of Instruction

- 1) Lecture and discussion
- 2) Teamwork

Out-of-Class Assignments

- 1) **Reading:** Lecture notes; assigned textbook sections explaining mathematical concepts and procedures; instructional materials provided by the instructor; worked example sets; reference materials for mathematical symbols and notation; supplemental explanations from approved online resources; and documentation for graphing calculators or mathematical software.
- 2) **Writing:** Problem sets requiring clear, step-by-step demonstration of reasoning using correct mathematical notation; written explanations supporting solutions and justifying methods; error analysis and revision of incorrect or incomplete work; solution summaries explaining conceptual understanding; problem-based learning tasks requiring application of multiple skills; written responses to mathematical prompts or inquiry problems; and organized documentation of problem-solving strategies used in individual or group assignments.
- 3) **Other:** Online homework assignments or quizzes using learning platforms; collaborative problem solving using shared documents or discussion boards; graph construction by hand and with technology; review and annotation of instructor feedback to revise and resubmit work; preparation for exams through practice tests and concept reviews; mathematical modeling activities using real-world data; use of graphing calculators or math software to explore and verify solutions; and completion of study logs, learning reflections, or structured practice plans to build mastery.

Texts and References

- 1) Required (representative example): Stewart, James. *Multivariable Calculus*. 9th edition. Cengage, 2021.
- 2) Supplemental: None

Student Learning Outcomes

Upon successful completion of this course, students will be able to:

- 1) Apply analytical, numerical, and graphical methods to solve problems related to multi-variable functions.
- 2) Solve multi-disciplinary application problems and interpret the results in context.

*For the complete list of **learning objectives**, please see the **Course Objectives** section