Cheat Sheet – Final Exam

Derivatives

1. $\frac{d}{dx}(\tan x) = \sec^2 x$
2. $\frac{d}{dx}(\cot x) = -\csc^2 x$
3. $\frac{d}{dx}(\sec x) = \sec x \tan x$
4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
5. $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
6. $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
7. $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
8. $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$
9. $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$
10. $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$
11. $\frac{d}{dx}(\sinh x) = \cosh x$
12. $\frac{d}{dx}(\cosh x) = \sinh x$

Integrals

13. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
14. $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
15. $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
16. $\int \ln x\,dx = x\ln x - x + C$
17. $\int \tan x\,dx = \ln|\sec x| + C$
18. $\int \sec x\,dx = \ln|\sec x + \tan x| + C$
19. $\int \cot x\,dx = -\ln|\csc x| + C$
20. $\int \csc x\,dx = \ln|\csc x - \cot x| + C$
21. $\int \sec^3 x\,dx = \frac{\sec x \tan x}{2} + \frac{\ln|\sec x + \tan x|}{2} + C$
22. $\int \sec^2 x\,dx = \tan x + C$
23. $\int \csc^2 x\,dx = -\cot x + C$

Trig Identities

24. $\sin^2 x + \cos^2 x = 1$
25. $1 + \tan^2 x = \sec^2 x$
26. $1 + \cot^2 x = \csc^2 x$

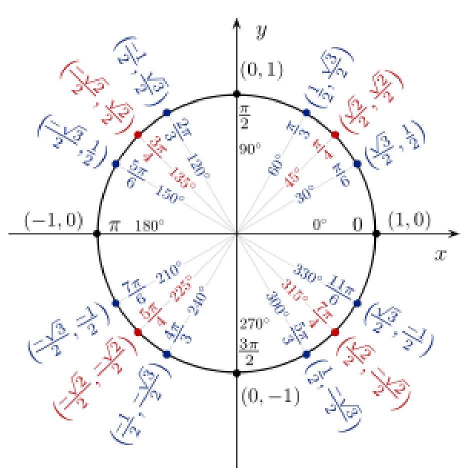
Right Angle Trigonometry

27. $\sin\theta = \frac{opp}{hyp}$ $\csc\theta = \frac{hyp}{opp}$
28. $\cos\theta = \frac{adj}{hyp}$ $\sec\theta = \frac{hyp}{adj}$
29. $\tan\theta = \frac{opp}{adj}$ $\cot\theta = \frac{adj}{opp}$

Half-Angle Formulas

30. $\sin^2 x = \frac{1}{2}(1-\cos 2x)$
31. $\cos^2 x = \frac{1}{2}(1+\cos 2x)$

	0	π/6	π/4	π/3	π/2
sin *x*	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos *x*	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
tan *x*	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Undef.



7.1 Integration by Parts

$$\int udv = uv - \int vdu$$

7.3 Trig Substitution

$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

Parametric Equations

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy/dt}{dx/dt}\right)}{dx/dt}$$

Area between curve and x-axis:

$$\int_\alpha^\beta ydx = \int_\alpha^\beta g(t)f'(t)dt$$

Arc Length: $L = \int_\alpha^\beta \sqrt{\left(dx/dt\right)^2 + \left(dy/dt\right)^2}\,dt$

Polar Equations

$x = r\cos\theta \qquad r^2 = x^2 + y^2$

$y = r\sin\theta \qquad \tan\theta = \frac{y}{x}$

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

Area inside the curve: $\int_\alpha^\beta \frac{1}{2}r^2 d\theta$

Area between curves: $\int_\alpha^\beta \frac{1}{2}\left(r_1^2 - r_2^2\right)d\theta$

Arc Length: $L = \int_\alpha^\beta \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\,d\theta$

Function

Arc Length: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

L'Hospital's Rule:

If $\frac{f(x)}{g(x)} \to \frac{0}{0}$ or $\frac{f(x)}{g(x)} \to \frac{\infty}{\infty}$, then

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{f'(x)}{g'(x)}$$

11.3 Error bounds for Integral Test

If *f* is continuous, positive and decreasing, then the error is bounded by:

$$\int_{n+1}^\infty f(x)dx \le R_n \le \int_n^\infty f(x)dx \text{ and}$$

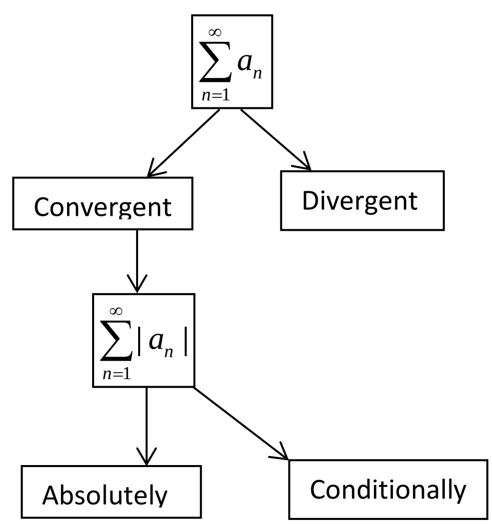
$$S_n + \int_{n+1}^\infty f(x)dx \le S \le S_n + \int_n^\infty f(x)dx$$

11.5 Error bounds for Alternating Series Test

If $\sum_{n=1}^\infty (-1)^n b_n$ is an alternating series, then

$|R_n| \le b_{n+1}$ and $S_n \le S \le S_{n+1}$ or $S_{n+1} \le S \le S_n$

Absolute Convergence



11.10 Taylor and MacLaurin Series

If $f(x)$ has a power series centered at $x = a$, then

$$f(x)=\sum_{n=0}^{\infty} c_n (x-a)^n \qquad c_n=\frac{f^{(n)}(a)}{n!}$$

11.9 Writing Functions as a Power Series

$$\frac{a}{1-r}=\sum_{n=0}^{\infty} ar^n$$

Exponent Rules

$$\left(\frac{ab}{c}\right)^n=\frac{a^n b^n}{c^n}$$

$$a^m a^n = a^{m+n}$$

Known Power Series

$$\frac{1}{1-x}=\sum_{n=0}^{\infty} x^n$$

$$\frac{1}{1+x^2}=\sum_{n=0}^{\infty}(-1)x^{2n}$$

$$\tan^{-1} x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{2n+1}$$

$$\ln(1+x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}$$

$$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

Test	Series	Convergence or Diverges	Comments
Divergence test	$\sum a_n$	Diverges if $\lim_{n\to\infty} a_n \neq 0$	Inconclusive if $\lim_{n\to\infty} a_n = 0$
Geometric series	$\sum_{n=0}^{\infty} ar^n$ or $\sum_{n=1}^{\infty} ar^{n-1}$	Converges to $\frac{a}{1-x}$ only if $\lvert r\rvert < 1$ Diverges if $\lvert r\rvert \geq 1$	Useful for comparison tests if the n^{th} term a_n of a series is similar to ax^n.
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$ Diverges if $p \leq 1$	Useful for comparison tests if the n^{th} term a_n of a series is similar to $\frac{1}{n^p}$.
Integral	$\sum_{n=c}^{\infty} a_n \quad (c \geq 0)$ $a_n = f(n)\ for\ all\ n$	Converges if $\int_c^{\infty} f(x)\,dx\ converges$ Diverges if $\int_c^{\infty} f(x)\,dx\ diverges$	The function *f* obtained from $a_n = f(n)$ must be continuous, positive, decreasing and readily integrable for $x \geq c$.
Comparison	$\sum a_n$ and $\sum b_n$	If $\sum b_n$ converges and $0 \leq a_n \leq b_n$, then $\sum a_n$ converges If $\sum b_n$ diverges and $0 \leq b_n \leq a_n$, then $\sum a_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a *p*-series.
Limit comparison	$\sum a_n$ and $\sum b_n$ With $a_n, b_n > 0$ for all n And $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$	$\sum b_n$ converges $\rightarrow \sum a_n$ converges $\sum b_n$ diverges $\rightarrow \sum a_n$ diverges	The comparison series $\sum b_n$ is often a geometric series or a *p*-series. To find b_n consider only the terms of a_n that have the greatest effect on the magnitude.
Ratio	$\sum a_n$ with $\lim_{n\to\infty} \frac{\lvert a_n+1\rvert}{\lvert a_n\rvert} = L$	Converges (absolutely) if L< 1 Diverges if L > 1 or if L is infinite	Inconclusive if L = 1. Useful if a_n involves factorials or n^{th} powers.
Root	$\sum a_n$ with $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L$	Converges (absolutely) if L< 1 Diverges if L > 1 or if L is infinite	Test is Inconclusive if L = 1. Useful if a_n involves n^{th} powers.
Absolute Value $\sum \lvert a_n\rvert$	$\sum a_n$	$\sum \lvert a_n\rvert$ converges $\rightarrow \sum a_n$ converges	Useful for series containing both positive and negative terms.
Alternating series	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n \quad (b_n > 0)$	Converges if $0 > b_{n+1} < a_n$ for all n and $\lim_{n\to\infty} b_n = 0$	Applicable only to series with alternating terms.