{Principle Concepts on Matrices}

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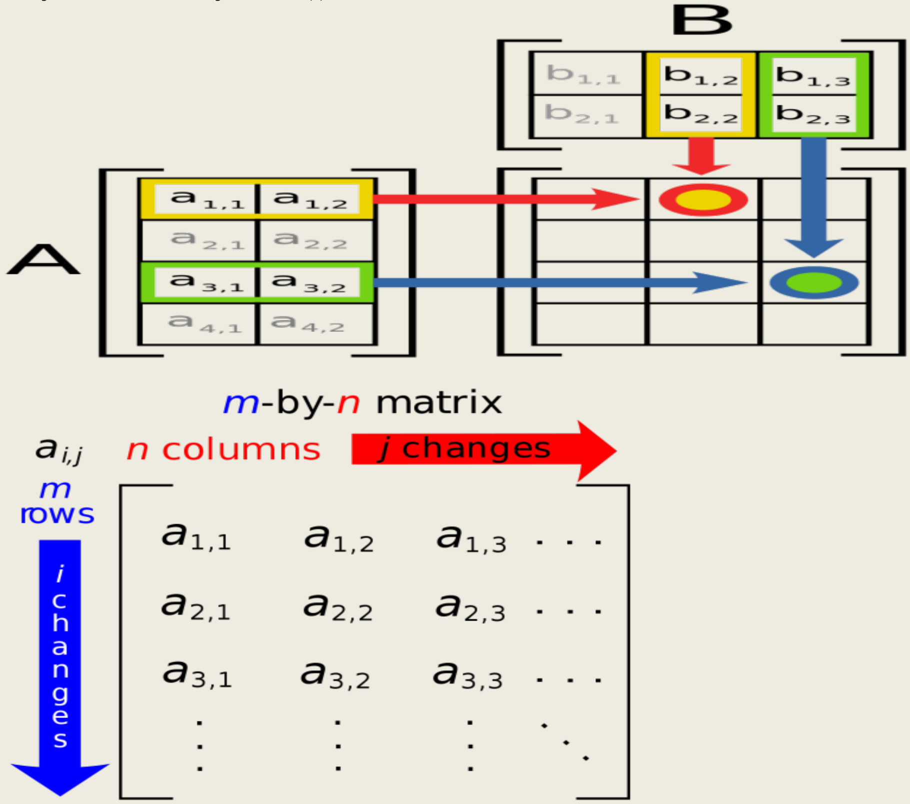
Definition:

A rectangular array of m x n numbers (real or complex) in the form of m horizontal lines (called rows) and n vertical lines (called columns), is called a *matrix* of order m by n, written as m x n matrix. Such an array is enclosed by [ ] or ( ).



Some Types of Matrices:

1 – Row Matrix, A = $[a_{ij}]_{1 \times n}$

A = [1 2 4 5] is a row matrix of order 1 x 4.

2 - Column Matrix, A = $[a_{ij}]_{m\times1}$

B = $\begin{bmatrix} 7 \\ -4 \\ 0 \end{bmatrix}$ is a column matrix of order 3 x 1.

3 - Zero or Null Matrix, A = $[a_{ij}]_{mxn}$ where, $a_{ij} = 0$

O = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ is a zero matrix of order 3 x 2.

4 - Singleton Matrix, A = $[a_{ij}]_{mxn}$ where, m = n =1

C = [5]

5 - Horizontal Matrix, $[a_{ij}]_{mxn}$ where, n > m,

H = $\begin{bmatrix} 6 & -4 & 11 \\ 5 & 0 & -3 \end{bmatrix}$, H is matrix of order 2x3.

6 - Vertical Matrix, $[a_{ij}]_{mxn}$ where, m > n,

V = $\begin{bmatrix} 4 & 1 \\ 13 & 9 \\ -5 & 3 \end{bmatrix}$, V is a matrix of order 3x2.

7 - Square Matrix, $[a_{ij}]_{mxn}$ where, m = n,

S = $\begin{bmatrix} 1 & 0 & 23 \\ -9 & 0.4 & 7 \\ 8 & 4 & 11 \end{bmatrix}$, S is a matrix of order 3x3.

8 - Diagonal Matrix, A = $[a_{ij}]$ where, a_{ij} = $\{0, i \neq j\}$,

D = $\begin{bmatrix} 15 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 18 \end{bmatrix}$, A is a matrix of order 3x3.

9 - Scalar Matrix, A = $[a_{ij}]_{mxn}$ where, a_{ij} = $\begin{Bmatrix} 0, & i \neq j \\ k, & i = j \end{Bmatrix}$,

K = $\begin{bmatrix} 19 & 0 & 0 \\ 0 & 19 & 0 \\ 0 & 0 & 19 \end{bmatrix}$, K is a matrix of order 3x3.

10- Identity (Unit) Matrix, A = $[a_{ij}]_{m\times n}$ where, $a_{ij} = \begin{Bmatrix} 1, & i = j \\ 0, & i \neq j \end{Bmatrix}$,

I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ OR $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, I is a matrix of order 2x2.

11- Triangular Matrices, Can be either upper triangular ($a_{ij} = 0$, when i > j) or lower triangular ($a_{ij} = 0$ when i < j),

U = $\begin{bmatrix} 1 & 60 & 3 \\ 0 & -4 & 0 \\ 0 & 0 & 12 \end{bmatrix}$, L = $\begin{bmatrix} -4 & 0 & 0 \\ 1 & 23 & 0 \\ 15 & -3 & 0.33 \end{bmatrix}$

OPERATIONS ON MATRICES

The four "basic operations" on numbers are addition, subtraction, multiplication, and division. For matrices, there are basic operations with rows and the matrices themselves; the procedures that you can do with the rows of a matrix are *row-switching* and *multiplying row by a scalar*, while basically, the operations on matrices are, *add/subtract matrices* ,*multiply a matrix by constant* (scalar multiple) and *multiplying two matrices*.

Row-switching:

For instance, given the matrix A = $\begin{bmatrix} 0 & 0 & 1 & 3 \\ 1 & 2 & 3 & 5 \\ 0 & 1 & -2 & 4 \end{bmatrix}$.

You can switch the rows around to put the matrix into a nicer row arrangement, like this:

A = $\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$,

When switching rows around, be careful to copy the entries correctly.

Multiplying row by a scalar:

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 0 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 \times 3 & 2 \times 5 \\ 2 \times 1 & 2 \times 0 \end{bmatrix}$$

$$2A = \begin{bmatrix} 6 & 10 \\ 2 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 7 & 0 & 4 \\ 2 & -3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

Note: The notation $2A$ means to multiply matrix A by 2.

$$2A = \begin{bmatrix} 2\times7 & 2\times0 & 2\times4 \\ 2\times2 & 2\times-3 & 2\times0 \\ 2\times1 & 2\times1 & 2\times-1 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 8 \\ 4 & -6 & 0 \\ 2 & 2 & -2 \end{bmatrix}$$

Every entry of A is multiplied by 2. final answer

Adding / subtracting two matrices:

When you add / subtract two matrices they have to be of the same order

That is $A \pm B$ must be, A_{mxn} and B_{mxn}

$$\begin{bmatrix} -4 & 3 \\ 1 & 9 \end{bmatrix} + \begin{bmatrix} 6 & 1 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} -4+6 & 3+1 \\ 1+8 & 9+(-2) \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 9 & 7 \end{bmatrix}$$

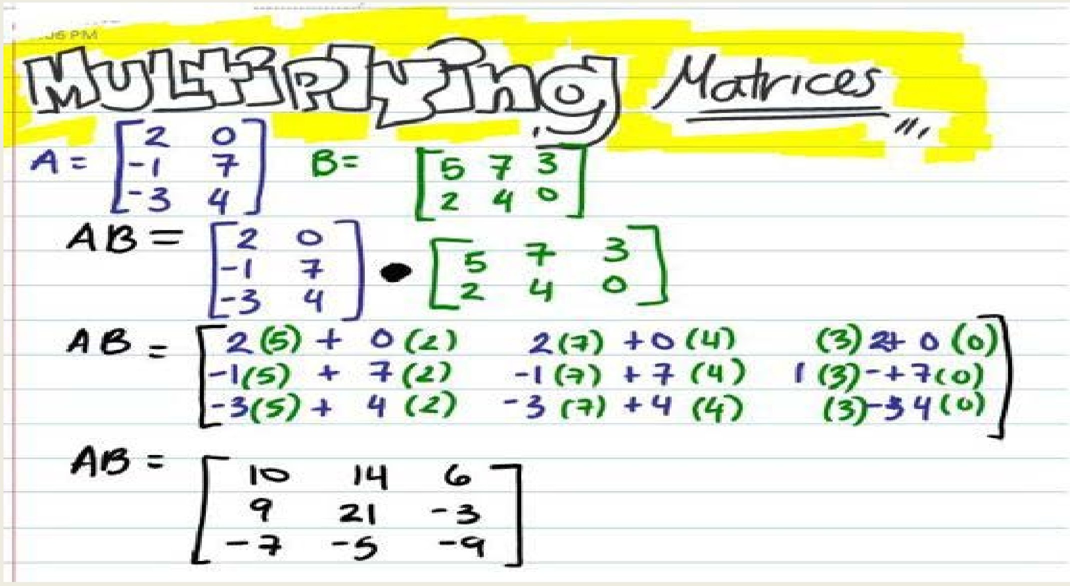
final answer

$3-4=-1$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

Multiplying two matrices:

When you multiply two matrices A and B they have to be of this type of order, A_{mxn} , B_{nxm}



Properties of Matrix Multiplication

Let A, B and C be three matrices, then

- $AB \neq BA$
- $(AB)C = A(BC)$
- $A.(B+C) = A.B+A.C$
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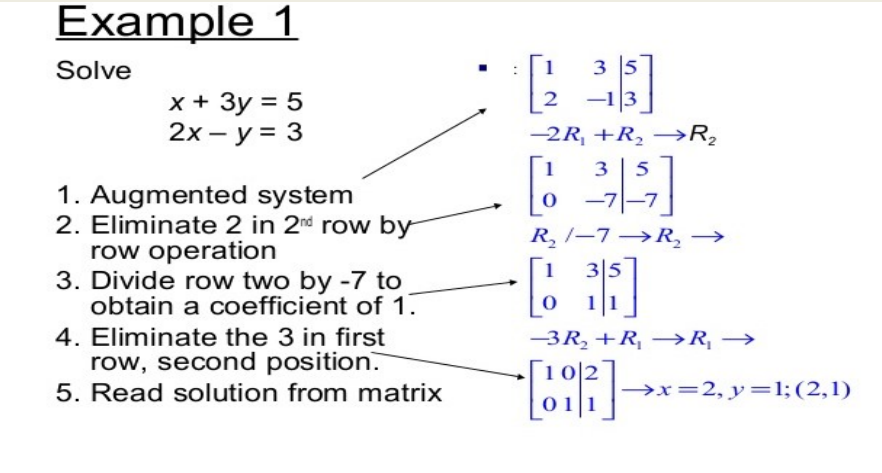
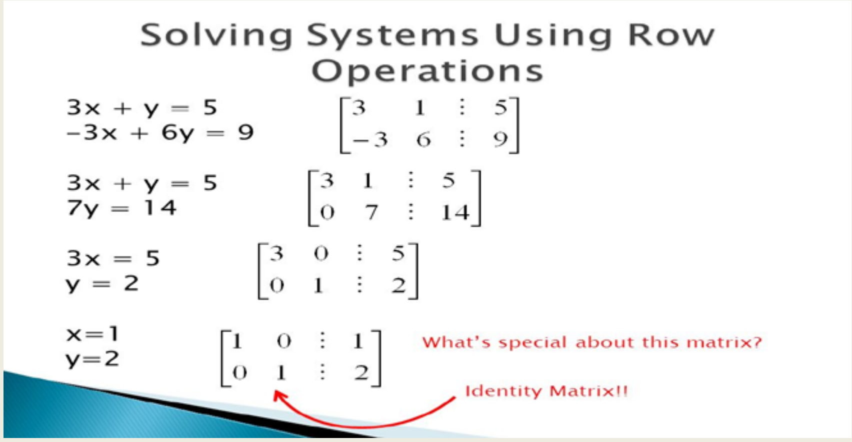
Use Matrices to Solve a System of Linear Equations:

One of the most use of matrices (especially for those a Math. 110/10 and 110 students), is to solve a system of linear questions, using the row operations given above.



Example 2


Solving Systems Using Row Operations
3x + y = 5
−3x + 6y = 9
3x + y = 5
7y = 14
3x = 5
y = 2
x=1
y=2
What's special about this matrix?
Identity Matrix!!