Solving Systems of Equations with Three Variables

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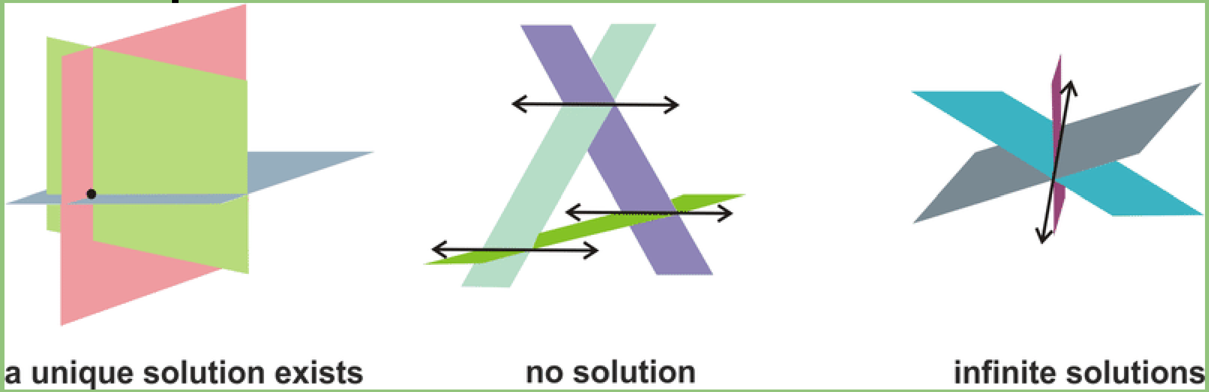
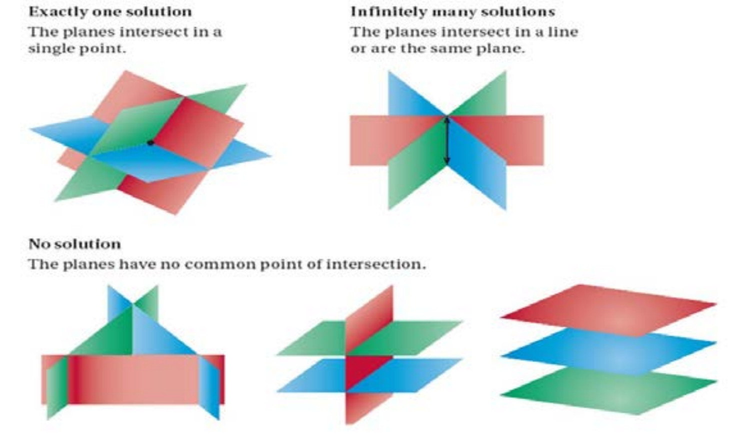
Describing Solutions to a System of Three Equations in Three Variables

- Ax+By+Cz=D
- Each equation defines a flat plane that can be graphed on a 3D x-y-z graph.
- The solution is when these three planes cross a single point.
- Another type of solution has an infinite number of points: a three dimensional straight line.
- To solve for single point solutions, we can use Elimination or Substitution.
- No solution occurs in some systems such as parallel or triangular planes.

Visualizing Solutions to a System of Three Equations in Three Variables





Solving a System of Equations Algebraically

To solve a system of three equations in three variables, we will be using the linear combination method. This time we will take two equations at a time to eliminate one variable and using the resulting equations in two variables to eliminate a second variable and solve for the third.

Example:

$$x-3y+3z=-4$$

$$2x+3y-z=15$$

$$4x-3y-z=19$$

Solution:

1) Pair equations to eliminate 1 variable

$$\begin{array}{l} x - 3y + 3z = -4 \\ 2x + 3y - z = 15 \\ \hline 3x + 2z = 11 \end{array} \qquad \begin{array}{l} 2x + 3y - z = 15 \\ 4x - 3y - z = 19 \\ \hline 6x - 2z = 34 \end{array}$$

2) Solve new system

$$\begin{array}{l} 3x + 2z = 11 \\ 6x - 2z = 34 \\ \hline 9x = 45 \\ x = 5 \end{array}$$

$$3x + 2z = 11$$
$$15 + 2z = 11$$
$$2z = -4$$
$$z = -2$$

$$2x + 3y - z = 15$$
$$2(5) + 3y - (-2) = 15$$
$$y = 1$$

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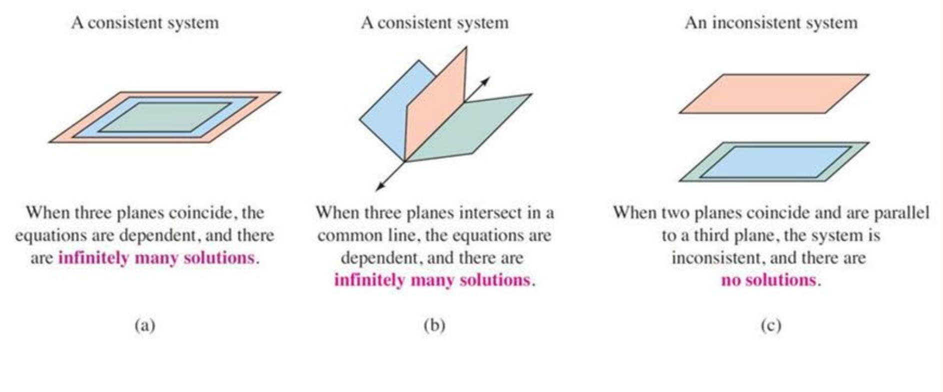
Solution is (5,1, -2)

Identifying Inconsistent Systems and Dependent Equations

- When the equations in a system of two equations with two variables are dependent, the system has infinitely many solutions
  - This is NOT always true for systems of three equations with three variables.
    - A system can have dependent equations and still be inconsistent in this case.
- The illustration demonstrates the different possibilities



Using a Matrix to Solve a System of Equations

- Step 1: Write the coefficients in a matrix using a vertical line to represent equals signs.
- Step 2: Find the inverse of the matrix that’s left of the equals signs.
- Step 3: Multiply the inverse matrix by the part of the matrix that is right of the equals sign.

Example of Using a Matrix

$x + y - z = -2$

$2x - y + z = 5$

$-x + 2y + 2z = 1$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

$$\xrightarrow{R1 + R3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ 0 & 3 & 1 & -1 \end{array}\right]$$

Example of Using a Matrix Continued

$$\xrightarrow{-2R1+R2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 3 & 1 & -1 \end{array}\right] \xrightarrow{R2 + R3} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & -3 & 3 & 9 \\ 0 & 0 & 4 & 8 \end{array}\right]$$

From the third row, 4z = 8.

To solve for z, divide both sides by 4, z= 2.

From the second row, -3y + 3z = 9. Substitute z=2, -3y + 6 = 9. Subtract 6 on both sides= -3y = 3.
Divide -3 on both sides, y = -1.

From the first row, x + y - z = -2. Solving for x, substitue y= -1 and z= 2.
x - 1 - 2 = -2.
x -3 = -2. Add three to both sides, x= 1.
The solution is (1, -1, 2).