

5.1 GCF and Factoring by Groups

■ Need To Know

- Definitions
- How to factor by GCF
- How to factor by groups



The Greatest Common Factor

Definition - _____ means to write a number as product of smaller numbers.

Example:

Definition - _____ a polynomial means to write a polynomial as a product of smaller polynomials.

Example:



The Greatest Common Factor

Definition – The _____
for a polynomial is the largest monomial that
divides each term of the polynomial.

Examples – Find the GCF of each:

$$5x + 5$$

$$10x^6 + 15x^4$$

$$8a^6 + 12a^4 - 4a^3$$



The Greatest Common Factor

Factor each polynomial

$$ab + ac$$

$$5x + 5$$

$$10x^6 + 15x^4$$

$$8a^6 + 12a^4 - 4a^3$$



Practice

Factor each polynomial (you try)

$$25x^4 + 35x^3$$

$$20x^8 - 12x^7 + 4x^6$$



GCF's may not be Monomials

Factor:

$$x(x + a) + 2(x + a)$$



Factoring by Groups

Factor:

$$a^3 - 7a^2 + 4a - 28$$

Factor by Groups

1. No GCF and 4 terms
2. Factor GCF's from 2 groups of two
3. Factor binomial GCF from groups



Factoring by Groups

Factor by Groups :

$$8x^3 + 12x^2 - 14x - 21$$

Factor by Groups

1. No GCF and 4 terms
2. Factor GCF's from 2 groups of two
3. Factor binomial GCF from groups

end

5.2 Factoring Trinomials

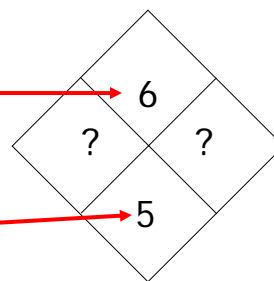
- Need To Know
 - Diamond puzzle
 - Idea of reverse FOIL
 - Factoring Trinomials



Diamond Puzzle

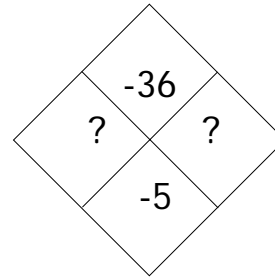
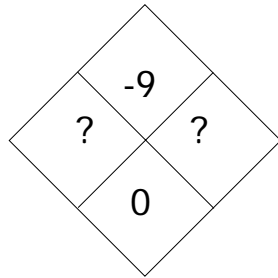
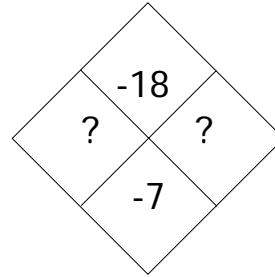
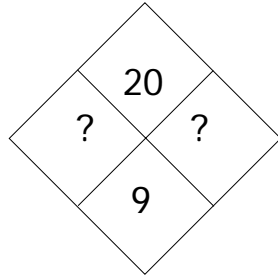
This number skill game builds the mental skills you need to become good at factoring.

Find two numbers that
multiply to the top number
and
add to the bottom number.





Practice




The Foil Method in Reverse

Multiply:

$$(z - 4)(z + 5)$$

Short Cut: **FOIL**

F – first terms mult.

O – outer terms mult.

I – inner terms mult.

L – last terms mult.



Factoring Trinomials

Factor:

$$x^2 + 5x + 6$$

Factor:

$$z^2 - 7z + 6$$



Factoring Trinomials w/ GCF

Factor:

$$2a^2 - 2a - 24$$

Factor:

$$3z^3 + 18z^2 + 15z$$



Factoring Trinomials w/ 2 Vars

Factor:

$$x^2 + 7xy + 12y^2$$

Factor:

$$z^2 - 3za - 10a^2$$

end



5.3 More Factoring Trinomials

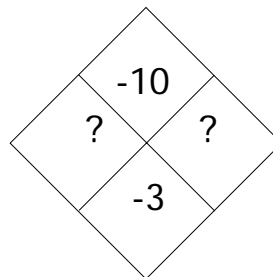
- Need To Know
 - Review diamond puzzle
 - Methods of factoring Trinomials
 1. Guess, check and revise
 2. The Grouping Method
(see the book for this technique)



Recall Diamond Puzzle

Factor:

$$x^2 - 3x - 10$$





Factoring Trinomials w/ $a \neq 1$

Factor:

$$2a^2 + 7a + 6$$

Standard Form:

$$ax^2 + bx + c$$

Guesses – All possible options:

$$(2a + 6)(a + 1) \quad (2a + 1)(a + 6) \quad (2a + 3)(a + 2) \quad (2a + 2)(a + 3)$$

$$(2a - 6)(a - 1) \quad (2a - 1)(a - 6) \quad (2a - 3)(a - 2) \quad (2a - 2)(a - 3)$$

$$(2a + 6)(a - 1) \quad (2a + 1)(a - 6) \quad (2a + 3)(a - 2) \quad (2a + 2)(a - 3)$$

$$(2a - 6)(a + 1) \quad (2a - 1)(a + 6) \quad (2a - 3)(a + 2) \quad (2a - 2)(a + 3)$$



Factoring Trinomials w/ $a \neq 1$

Factor:

$$4x^2 - x - 5$$



Factoring Trinomials w/ $a \neq 1$

Factor:

$$30y^2 + 5y - 5$$



Factoring Trinomials w/ $a \neq 1$

Factor:

$$15t^2 - 2t - 8$$



Factoring Trinomials w/ $a \neq 1$

Factor:

$$10p^2 + 5pq - 30q^2$$



5.3 Conclusion

Ways to Factor Polynomial

1. By Greatest Common Factor (GCF)
2. By Grouping
3. Factor Trinomials
 - Guess, check and revise
 - The Group Method (see book)

end



5.4 Special Factoring

- Need To Know

- Recall some formulas
- Factoring the difference of two squares
- Factoring perfect square trinomials



Formulas From Poly Multiplication

Formulas to Remember

$$(x + y)(x - y) = x^2 - y^2$$

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$



Factoring a Difference of Squares

Factor:

$$9x^2 - 25$$

Formulas to Know

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$



Factoring a Difference of Squares

Factor:

$$49x^2 - 1$$

Formulas to Know

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$



Factoring a Difference of Squares

Factor:

$$x^2 + 25$$

Formulas to Know

$$x^2 - y^2 = (x + y)(x - y)$$

$x^2 + y^2$ is prime

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$



Factoring a Difference of Squares

Factor:

$$a^4 - b^4$$

Formulas to Know

$$x^2 - y^2 = (x + y)(x - y)$$

$x^2 + y^2$ is prime

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$



Factoring Perfect Square Trinom.

Factor:

$$m^2 + 12m + 36$$

Formulas to Know

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + y^2 \text{ is prime}$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$



Factoring Perfect Square Trinom.

Factor:

$$4x^2 + 12x + 9$$

Formulas to Know

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + y^2 \text{ is prime}$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$



Factoring Perfect Square Trinom.

Factor:

$$6x^2 - 24x + 24$$

Formulas to Know

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^2 + y^2 \text{ is prime}$$

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$



5.4 Conclusion

Ways to Factor Polynomial

1. By Greatest Common Factor (GCF)
2. By Grouping
3. Factor Trinomials
 - Guess, check and revise
4. Factoring with Formulas

- $x^2 - y^2 = (x + y)(x - y)$

- $x^2 + y^2$ is prime

- $x^2 + 2xy + y^2 = (x + y)^2$

- $x^2 - 2xy + y^2 = (x - y)^2$

5.5 Factoring Review

- Need To Know
 - Factoring Review
 - Practice



Ways to Factor Based on Terms

- A. Check for GCF on all factoring (Always do first!)
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 -
 -
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas:
 - 4. Four Terms
 -
- C. Always Factor Completely



Factoring Practice

Factor:

$$3x^4 - 27x^2$$

WAYS TO FACTOR

- A. Factor GCF
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely



Factoring Practice

Factor:

$$2x^5 + 20x^4 + 50x^3$$

WAYS TO FACTOR

- A. Factor GCF
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely



Factoring Practice

Factor:

$$y^5 + 36y^3$$

WAYS TO FACTOR

- A. Factor GCF
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely



Factoring Practice

Factor:

$$15a^2 - a - 2$$

WAYS TO FACTOR

- A. Factor GCF
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely



Factoring Practice

Factor:

$$4x^4 - 12x^3 - 40x^2$$

WAYS TO FACTOR

- A. Factor GCF
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely



Factoring Practice

Factor:

$$2a^5b + 6a^4b + 2a^3b$$

WAYS TO FACTOR

- A. Factor GCF
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely



Factoring Practice

Factor:

$$3ab + 9a + 2b + 6$$

WAYS TO FACTOR

- A. Factor GCF
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas
 - $x^2 + 2xy + y^2 = (x + y)^2$
 - $x^2 - 2xy + y^2 = (x - y)^2$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely



5.5 Conclusion

- A. Check for GCF on all factoring (Always do first!)
- B. Look at the Number of Terms
 - 2. Two Terms – Formulas
 - $x^2 - y^2 = (x + y)(x - y)$
 - $x^2 + y^2$ is prime
 - 3. Three Terms
 - Guess, check, and revise
 - Formulas: $x^2 + 2xy + y^2 = (x + y)^2$
 $x^2 - 2xy + y^2 = (x - y)^2$
 - 4. Four Terms
 - By Grouping
- C. Always Factor Completely

5.6 Solving Quadratic Equation

- Need To Know

- Vocabulary and facts
- Solving quadratic equations



Vocabulary

Definition – a **q**_____ is one that can be put in the form $ax^2 + bx + c = 0$ where a, b, & c are real numb. ($a \neq 0$).

$ax^2 + bx + c = 0$ is a quadratic in **s**_____.

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| | |
Quadratic Term Linear Term Constant Term

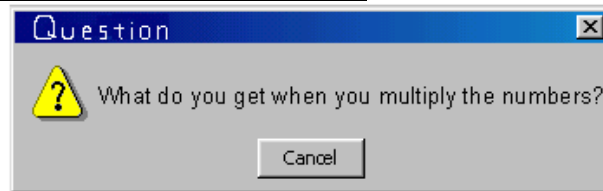
Vocabulary

Definition – The **d**_____ of a polynomial in one variable is the exponent from the term with the highest power.

Fact – The degree of an equation indicates the _____.

The Principle of Zero Products

Illustrate the Main Idea



The Principle of Zero Products

For real numbers a and b,

if $a \cdot b = 0$, then _____



Solve Quadratic Equations

$$x^2 + 6x + 8 = 0$$

Steps to Solve Q.E.

1. Set up must = 0
2. Factor completely
3. Solve each factor for zero
4. Check solutions



Solve Quadratic Equations

Solve for x:

$$2x^2 + 5x = 3$$

Steps to Solve Q.E.

1. Set up must = 0
2. Factor completely
3. Solve each factor for zero
4. Check solutions



Solve Quadratic Equations

Solve for a:

$$49a^2 - 16 = 0$$

Solve for z:

$$5z^2 = -2z$$



Solve Quadratic Equations

Solve for w:

$$w(w - 13) = -40$$

Solve for x:

$$(x + 3)(3x + 5) = 7$$

end

5.7 Applications

■ Need To Know

- Recall guide lines to solve word problems
- Recall tool to solve word problems
- Solve word problems with 2 unknowns



Guide Lines to Solve Problems

The author call these a blueprint for solving

1. Read and understand the problem (# of unknowns)
2. Assign variables and write down the meaning of the variable
3. Write an equation
4. Solve the equation
5. Write down your answer using a complete sentence
6. Reread and check your solution



Tools for Solving Problems

These tools are things you can do when you are stuck on a word problem.

1. Use keywords
2. Draw a picture
3. Make up a simpler problem
4. Make tables of numbers and look for patterns
5. Use charts to organize your information
6. Use a verbal model



Apply

A number is two less than its square.
Find all such numbers.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model



Apply

The product of two consecutive odd integers is 63. Find the integers.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model



Apply

The length of a rectangle is 2 more than twice the width. The area is 60 in^2 . Find the dimensions.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model



Apply

The short leg of a triangle is 2cm shorter than the long leg. The hypotenuse is 10cm. Find the sides.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model



Apply

If a water rocket is launched with an initial velocity of 48 ft/sec, its height, h , in feet, after t seconds, is given by $h = 48t - 16t^2$. Find the height of the rocket after 1.5 seconds.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model